Soluzioni tutorato3

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Esercizio 1. (i)$y' = 5x^4 - 12x^2 + 2$.

(ii)$y = x^2\sqrt[3]{x^2} = x^2x^{2/3} = x^{8/3}$, quindi $y' = (8/3)x^{5/3}$.

(iii)$y' = \frac{2(x^2-5x+5)-(2x+3)(2x-5)}{(x^2-5x+5)^2} = \frac{2x^2-10x+10-(4x^2-10x+6x-15)}{(x^2-5x+5)^2} = \frac{-2x^2-6x+25}{(x^2-5x+5)^2}$.

(iv)$y' = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\sin^2 x\cos^2 x}$.

(v)$y' = \frac{1}{2}(2x\arctan x + \frac{1}{1+x^2}(1+x^2) - 1) = x\arctan x$.

(vi)$y' = \frac{5x^4e^x - e^xx^5}{e^{2x}} = \frac{5x^4-x^5}{e^x}$.

(vii)$y' = e^x\arcsin x + e^x\frac{1}{\sqrt{1-x^2}}$.

(viii)$y' = \frac{2x\ln x - \frac{1}{x}x^2}{\ln^2(x)} = \frac{x(2\ln x-1)}{\ln^2 x}$.

(ix)$y' = \frac{1}{x}\log_{10}x + \frac{\ln x}{x}\log_{10}e - \ln a\frac{1}{x}\log_a e = \frac{1}{x}\log_{10}x + \frac{\ln x}{x\ln 10} - \frac{1}{x} = 2\frac{\ln x}{x\ln 10} - \frac{1}{x}$ dove abbiamo usato il fatto che $\log_a b = \frac{\ln b}{\ln a}$.

(x)$y' = \frac{1}{3}(\sin^2 x)^{-\frac{2}{3}}2\sin x\cos x + 3\cos^{-4}x\sin x = \frac{2}{3}\frac{\cos x}{\sqrt[3]{\sin x}} + 3\frac{\sin x}{\cos^4 x}$.

(xi)$y' = 3\cos(3x) - \frac{1}{5}\sin\frac{x}{5} + \frac{1}{2}\frac{1}{\sqrt{x}\cos^2(\sqrt{x})}$.

(xii)$y' = \frac{3}{4}\frac{x^2-1}{x^2+1}\frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2} + \frac{1}{4}\frac{x+1}{x-1}\frac{x+1-x+1}{(x+1)^2} + \frac{1}{2}\frac{1}{1+x^2} = \frac{-3x}{(x^2+1)(x^2-1)} + \frac{1}{2}\frac{1}{(x-1)(x+1)} + \frac{1}{2}\frac{1}{x^2+1} = \frac{-6x+x^2+1+x^2-1}{2(x^2+1)(x+1)(x-1)} = \frac{x^2-3x}{(x^4-1)}$

(xiii)$y' = \frac{e^x-e^{-x}}{2} = sh(x)$

(xiv)$y' = 3^xlog3$

(xv)$y' = 2cosx$

(xvi)$y' = cos(x^2+1)2x$

(xvii)$y' = \frac{1}{x^2-5}2x$

(xviii)$y' = e^{cos2x+sen2x}(-2sen2x+2cos2x)$

(xix)$y' = sh(\sqrt{senx})\frac{1}{2}senx^{1/2-1}cosx = \frac{sh(\sqrt{senx})cosx}{2\sqrt{senx}}$

(xx)$y' = \frac{2x^3senx-2xe^xsenx+x^4cosx-x^2e^xcosx-2x^3e^xsenx-x^4e^xsenx+x^2e^xsenx}{(x^2-1)^2e^{2x}} = \frac{x^4(cosx-e^xsenx)+2x^3(senx-e^xsenx)+x^2(e^xsenx-e^xcosx)-2xe^xsenx}{(x^2-1)^2e^{2x}}$

(xxi)$y' = \frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

(xxii)$y' = \frac{1}{x-\sqrt{x^2-3}}(1-\frac{x}{\sqrt{x^2-3}})$

(xxiii)$y' = 6cosx - \frac{1}{senx+\sqrt{sen^2x-1/4}}(cosx+\frac{2senxcosx}{2\sqrt{sen^2x-1/4}})$

(xxiv)se $x>0$ $y = e^{arctan(\frac{1+x}{1-x})-1/2ln(1+x^2)}$ e $y' = e^{arctan(\frac{1+x}{1-x})-1/2ln(1+x^2)}(\frac{1-x}{1+x^2})$,se $x<0$ $y = e^{\pi/4-ln(1-x)}$ e $y' = \frac{e^{\pi/4-ln(1-x)}}{1-x}$

Esercizio 2. $lim_{h\to 0}\frac{sen(x+h)-senx}{h} = lim_{h\to 0}2\frac{sen(h/2)cos(\frac{2x+h}{2})}{h} = lim_{h\to 0}cos(\frac{2x+h}{2}) = cosx.$

Esercizio 3. La pendenza di una curva é data dalla tangente dell'angolo formato dalla tangente alla curva nel punto in questione;quindi dalla derivata della funzione calcolata in quel punto;in questo caso la derivata é $y' = x/2$,che,calcolata nel punto é 1,a cui corrisponde un angolo di $\pi/4$.

Esercizio 4. (i)$y' = \frac{6a}{\sqrt{a^2+b^2}}x^5.$

(ii)$y' = -\frac{2}{3}ax^{-\frac{2}{3}-1} + b\frac{\sqrt[3]{x}+\frac{1}{3}x^{1-\frac{2}{3}}}{x^2\sqrt[3]{x^2}} = -\frac{2}{3}ax^{-\frac{5}{3}} + \frac{4}{3}bx^{-\frac{7}{3}} = \frac{4b}{3x^2\sqrt[3]{x}} - \frac{2a}{3x\sqrt[3]{x^2}}.$

(iii)$y' = \frac{(\cos x-\sin x)(\sin x-\cos x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2} = -\frac{2}{(\sin x-\cos x)^2}.$

(iv)$y' = 7x^6e^x + x^7e^x.$

(v)$y' = -\frac{1}{x^2} + 2\frac{1}{x} - \frac{1-\ln x}{x^2} = \frac{2}{x} - \frac{2}{x^2} + \frac{\ln x}{x^2}.$

(vi)$y' = 30(1+3x-5x^2)^{29}(3-10x).$

(vii)$y' = (2x-5)\cos(x^2-5x+1) - \frac{a}{x^2}\frac{1}{\cos^2\frac{a}{x}}.$

(viii)$y' = \frac{4}{3}(\frac{x-1}{x+2})^{-\frac{3}{4}}\frac{x+2-x+1}{(x+2)^2} = 4(x-1)^{-\frac{3}{4}}(x+2)^{-\frac{5}{4}}$.

(ix)$y' = 3cosx + 5senx$

(x)$y' = \frac{2x}{x^2-5}$

(xi)$y' = \frac{log_3 ecosx}{senx}$

(xii)$y' = \frac{2^x\sqrt{x}shxlog2log\sqrt{x}+\frac{2^x shx}{2\sqrt{x}}-\frac{2^x logxshx}{2\sqrt{x}}-2^x logx\sqrt{x}chx}{(shx)^2 x}$

(xiii)$y' = -\frac{e^x log(\sqrt{x^2+1})+\frac{e^x x}{x^2+1}}{e^{2x}log^2(\sqrt{x^2+1})}$