Soluzioni 10-AM3

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1. Facendo il cambio di coordinate

$$\begin{cases} x = ta\cos\varphi & \qquad 0 \le \varphi \le 2\pi \\ y = ta\sin\varphi & \qquad 0 \le t \le 1 \\ z = tb \end{cases}$$

e calcolando $d\sigma = at\sqrt{a^2+b^2}$, si ha che

$$\iint_S \sqrt{x^2+y^2}d\sigma = a^2\sqrt{a^2+b^2}\int_0^1 t^2\int_0^{2\pi} d\varphi\, dt = \frac{2}{3}\pi a^2\sqrt{a^2+b^2}.$$

2. Si parametrizzi la superficie Γ tramite

$$\varphi(t,u) = (t, u, 1-t-u)$$

da cui segue che $d\sigma = \sqrt{3}\, dudt$. Quindi si ha

$$\begin{aligned}
\int_\Gamma yz\sqrt{1-x}\, d\sigma &= \sqrt{3}\int_0^1\int_0^{1-t} u\sqrt{1-t}(1-t-u)dudt \\
&= \sqrt{3}\int_0^1\int_0^{1-t} u\sqrt{1-t}\, dudt - \sqrt{3}\int_0^1\int_0^{1-t} ut\sqrt{1-t}\, dudt \\
&\quad -\sqrt{3}\int_0^1\int_0^{1-t} u^2\sqrt{1-t}\, dudt \\
&= \sqrt{3}\int_0^1 \frac{(1-t)^{\frac{5}{2}}}{2}\, dt - \sqrt{3}\int_0^1\int_0^{1-t} t(1-t)^{\frac{5}{2}}\, dt \\
&\quad -\sqrt{3}\int_0^1 \frac{(1-t)^{\frac{7}{2}}}{2}\, dt \\
&= \left[\frac{\sqrt{3}}{7}(1-t)^{\frac{7}{2}} + 2\frac{\sqrt{3}}{7}t(1-t)^{\frac{7}{2}} + \frac{4\sqrt{3}}{63} - \frac{2\sqrt{3}}{9}(1-t)^{\frac{9}{2}}\right]_0^1
\end{aligned}$$

3. Passando in coordinate polari otteniamo

$$\begin{cases} x = \rho\cos\theta = e^{2\theta}\cos\theta \\ y = \rho\sin\theta = e^{2\theta}\sin\theta \end{cases}$$

Si ha inoltre

$$ds = \sqrt{x'(\theta)^2 + y'(\theta)^2}\, d\theta = \sqrt{5}\, e^{2\theta}\, d\theta$$

e sostituendo nell'integrale otteniamo

$$\int_\gamma (x^2 + y^2)^2\, ds = \sqrt{5}\int_{-\infty}^{0} e^{10\theta}\, d\theta = \frac{\sqrt{5}}{10}.$$

4. Si ha $ds = \sqrt{\dfrac{2 - t^2}{1 - t^2}}\, dt$ e dunque

$$\begin{aligned} \int_\gamma \cos x\, ds &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{2 - t^2}\, dt \\ &= 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^2 y\, dy \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 + \cos 2t)\, dt = \left[t + \frac{1}{2}\sin 2t\right]_{-\frac{1}{2}}^{\frac{1}{2}} \end{aligned}$$