Devlin's Angle

February 2008

Mathematics for the President and Congress

In last month's column, **American Mathematics in a Flat World**, I examined the implications of globalization for mathematics - how it is used in business, commerce, and society. As promised then, this month I take a look at the implications for how we teach mathematics - what gets taught and how.

As you will gather if you read my last piece, I believe that the entire mathematics education system needs to be rethought, particularly at the K-12 level. But that is a national issue, on which an individual teacher or college mathematics instructor has little influence. What college instructors and mathematics departments can do is design and give courses appropriate for the changing needs of society. In particular, courses that will prepare "non-quants" (graduates other than in mathematics, science, engineering, economics and finance, which includes the vast majority in Congress and the White House) for lives in the global economy.

In the globalized knowledge economy - the world of today, and increasingly of tomorrow - a good, but *appropriate*, knowledge of mathematics will be particularly crucial for those who run major businesses and the country. A CEO, a Member of Congress, or the President of the USA, does not need to be able to *do* mathematics. Others can do that for them. But in order to make informed decisions, they do need to have a *sound overall sense* of what mathematics can and cannot do, where it can be used and where not, and when to believe the figures and when to be skeptical.

To give one obvious and dramatic example of how lack of knowledge about mathematics can leave a decision maker at the mercy of individuals with an axe to grind or who are out to make a fast buck, I am moderately sure President Bush received "expert" advice on his much-taunted "No Child Left Behind" program, but a President who had sufficient understanding of mathematics would never have accepted such a misguided and disastrous policy, let alone promote it.

As I argued last time, in the digitally-connected global economy, we cannot win the competition for where much of the mathematics gets done any more than we were able to hang on to manufacturing, software development, or customer relations management. Increasingly, China (200,000,000 K-12 students) and India (211,000,000 K-12 students) will be where we send our mathematics to be done. (The US has a "mere" 53 million students, and we do a poor job of educating them mathematically; moreover, they have many choices of what to study and what career path to follow.) Our only strategy is to focus on what we do better than anyone else in the world: invent and innovate. And then we must make sure that we are able to control the products of that invention and innovation.

What we need to ensure is that all graduates from our schools and colleges have a knowledge of mathematics appropriate for this strategy.

I should stress that what I am talking about here is the *base* level of education. Our leading schools will continue to produce some of the most mathematically able people in the world. But just as it was once the case that *every* individual needed a mastery of arithmetic in order to function in society - an ability made redundant by the pocket-calculator, the computer, and the supermarket checkout machine - so too today's world requires a particular kind of mathematical knowledge.

Some of that need can be met by what are generally called "quantitative literacy" requirements across the curriculum, though few colleges have fully implemented such a requirement. Broadly speaking, QL encompasses a general sense of number and size, estimation skills, the ability to understand graphs, pie-charts and tables and to read a spreadsheet, the ability to reason logically and numerically, and a reasonable understanding of basic probability and statistics. Since the importance of such skills lies in their applications, QL should not be the subject of a course, which would surely fail to meet its goal, rather should be viewed as a requirement to be met across the entire curriculum.

QL is important in the "flat world" described by Thomas Friedman in his book **The World is Flat**. But the flat world creates a need for another kind of mathematical knowledge as well, one that I think is at least as important as quantitative literacy.

The goals of a mathematics course

A mathematics course can have each of the following goals:

1. To make students more aware of the nature and utility of mathematics, its breadth, its origins, its role in history, and its applications in modern society, including its relevance to their own lives.

2. To provide students with first-hand experiences of looking at the world through mathematical eyes and to ensure that they know what is involved in doing mathematics.

3. To make students competent in doing mathematics, both "pure math" and using mathematics to model and solve real world problems.

4. To make students achieve mastery in doing mathematics, both "pure math" and using mathematics to model and solve real world

problems.

5. To make students achieve mathematical proficiency, in the sense of the recommendations of the National Research Council's 1999-2000 Mathematics Learning Study Committee, described in their book *Adding it Up: Helping Children Learn Mathematics,* published by the National Academy Press in 2001.

6. To ensure that the students (and hence the school) perform well on a state mandated test that involves questions whose general template is known in advance. (The "No Child Left Behind" strategy.)

The first two goals focus on knowing about mathematics; goals 3 on 4 are about doing math; goal 5 is about becoming a "mathematician" (not necessarily in the professional sense, rather of learning how to think like a mathematician, which includes but goes well beyond mere technical competence in executing mathematical procedures); and the last goal is an institutional/political/financial one.

These days, a good school math course will have 3 or 4 as the goal, but a *typical* school math course has 6 as a major goal. (Sadly, when it comes to mathematics, good schools are the rarity.) From the perspective of the school, with a vested interest in its funding for the coming year, goal 6 is perfectly understandable; moreover, it is a goal that can be (and regularly is) achieved without meeting any of the other five goals, apart perhaps from a short-lived and superficial appearance of meeting goal 3!

But what goal or goals *should* a mathematics course have? The only people in favor of goal 6 are (i) politicians and government officials, whose real aim is often not education but getting elected (or re-elected), for which purpose they want figures that purport to demonstrate "improvement" in the math skills of the student population they oversee, and sometimes (ii) well meaning individuals outside the education system who think that education is a simple issue and convince themselves that they know better how to do it than the professionals. Neither group of individuals is ever likely to read anything that focuses on genuine educational issues, such as the MAA website, so from now on I shall drop any further consideration of goal 6, and consign it to the garbage can where it belongs.

Each of the remaining goals, 1 through 5, has merit, and it is possible to design an effective course that can (in principle, and to some extent) meet any one of those goals. Of these, goal 5 is surely the one most obviously worthy of achieving, not least because it implies most of the others (including goal 6 as it happens). Since there is a clear implication chain

> 5 implies 4 implies 3,

goal 5 subsumes goals 3 and 4, and since

> 5 implies 2,

achieving goal 5 will in fact automatically hit all of goals 2, 3, and 4.

This is in essence why the NRC committee made mathematical proficiency their main goal, although they did not articulate it in the fashion I just have.

College and university mathematics courses designed for science, engineering, or business majors tend to adopt a goal somewhere on the 3 - 4 - 5 spectrum. They want the student to learn how to apply mathematics in other domains, and how to solve the mathematical problems that arise in those domains.

College and university mathematics courses aimed at "nonscience" majors (the courses popularly labeled as "Math for Poets") generally take 3 as their primary goal and 2 as a secondary goal.

But what about goal 1?

This, I suggest, is the goal that is particularly important (as a basic requirement for all students) to the future of the US economy.

The home goal

Few courses have goal 1 as their primary aim - or even a secondary focus. Consequently, hardly any student graduates from a college or university with an awareness of the nature and utility of mathematics, its breadth, its origins, its role in history, and its applications in modern society, including its relevance to their own lives (goal 1), and only a few students graduate having experienced looking at the world through mathematical eyes and knowing what is involved (in the broad sense) in doing mathematics (goal 2). This is true even for mathematics majors.

There are two good reasons why ignoring goal 1 is not a desirable state of affairs.

First, modern life is heavily dominated by mathematics - albeit mostly behind the scenes - and much that goes on in the world can be properly followed and understood only by citizens who do have a good general awareness of mathematics. Consequently, an individual who graduates from school or university without such knowledge is severely disadvantaged when it comes to contributing to society.

Many are so significantly disadvantaged that, like the proverbial blind man who does not know he has a disability, they are not even aware of what they are missing. Our future as a nation will be at peril if such "mathematically blind" individuals become CEOs of major companies or are elected to Congress or to the White House.

Second, there is considerable evidence that student motivation is a major requirement for achieving any advancement toward any of goals 3, 4, and 5. In times past, self-interest, particularly financial self-interest, provided at least a modicum of motivation for people to master at the very least the elements of basic arithmetic. But in an era of ubiquitous computing devices, even that need has long since gone away. Students today are no less able or ambitious than previous generations, but they have many more choices of where and how to direct their attention and their efforts. They exhibit just as much effort and dedication to mastery of a domain that interests them as did their predecessors. But the key word here is "interest." When it comes to mathematics, few students have any interest.

The reasons for this lack of interest are not hard to find. The learning curve for mathematics is long and steep, with many opportunities to fall by the wayside. Without any real understanding of or appreciation of the human enterprise we call mathematics, few students are willing to devote sufficient time or effort to progress. They view mathematics as an ancient and arcane lore that long since stopped advancing, having little or no relevance to their lives. This at a time in history when mathematics is developing at a much faster rate than ever before, and has far great impact on society than in any previous era.

Achievement of goals 1 and 2 should not be left to an occasional, and often half-hearted and minimally supported, one-course "add-on" to a mathematics curriculum that focuses on one or more of goals 3, 4, and 5. In addition to their intrinsic value for non-science students, courses that focus on goals 1 and 2 should be viewed as an important attractor to, and precursor for, the other, more traditional mathematics courses! Motivate the student by showing them the elegance, power, and broad range of applications of modern mathematics, and many more will be inspired to make the effort required to achieve any of goals 3 to 5.

I propose the widespread introduction of courses that take goals 1 and 2 as their primary foci, and that we make the completion of such a course a graduation requirement.

How to score a home goal

Because such courses are so critical, both for the future of the nation and the future health of mathematics itself, when they are introduced they should not be handed off to the most junior instructors. They should be constructed with care. They should be given by the best and in general most senior mathematics faculty, who have the experience and breadth in the subject to give such courses well. And they should be allocated sufficient time.

That last point should not be overlooked. I have been giving such courses for almost twenty years now, on and off. (The current incarnation, Stanford's Math 15 course, is described at **www.stanford.edu/~kdevlin/courses08.html** .) I know from many years of experience that to give them well, they require at least twice the time commitment from the instructor as does a typical college mathematics course - not least because class support cannot in general be handed off to a graduate teaching assistant, who usually has neither the breadth of knowledge nor the teaching expertise to handle such an assignment adequately, let alone well.

With students who are mathematically averse (and you find such students even at elite institutions such as Stanford, Harvard, and Yale), there is another factor. Mathematical aversion can usually be traced back to one bad experience with a math teacher. The student's problem is not mathematics; it's a bad human-human experience. The only way to overcome that problem is to obliterate that bad human-human experience with a positive one, and to show the student you care. An instructor cannot hope to succeed in that regard if she or he hands off most of the class support activities to someone else, particularly to a graduate student - even if that graduate student is a stellar teaching assistant (and some are). To do so prevents the formation of a good instructor-student bond and sends the student entirely the wrong message.

Of course, college department chairs and deans will cry that they cannot afford such courses. Yes, I was a department chair for four years and a dean for eight, and I know the pressures under which chairs and deans have to operate. From the perspective of the chair or the dean's desk, it is usually true that the institution cannot afford to offer such courses. But from the perspective of national survival (as well as for the future health of our subject), we cannot afford not to.

Devlin's Angle is updated at the beginning of each month.

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