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Informational Inefficiency of Stock Prices under Asymmetric Information, Rational Expectation, and Imperfect Competition

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Contents

Introduction

This work aims to deepen the study of a celebrate risky asset pricing model in an incomplete market with asymmetrically informed risk-averse investors. This model was originally presented by Wang in his seminal paper [42] (1993) and extended by Monte, Perrotta & Fabretti in [32] (2010). Our goal is to study the impact on price formation of imperfect competition between asymmetrically informed risky averse investors, with rational expectations, in presence of non rational traders and market frictions. In this setting, we show the existence of equilibria in which the price of the risky asset, hereinafter *stock*, is caractherized by some extent of information inefficiency. More specifically, we find that the equilibrium stock price is semi-strong form efficient under low rational investors' risk aversion and low market noise volatility, while it loses efficiency as the rational investors' risk aversion or the market noise volatility increases. An interpretation intrigues us: as risk averse investors perceive a high exposure to market risk, defined in terms of investors' risk aversion and market noise volatility, they prefer to stress the strategic features of their trading, which rationally leads to informationally inefficient equilibrium prices.

The rational expectation equilibrium (REE) concept, developed by Lucas [30] (1972), Green [17] (1973), Grossman [18] (1976), and Kreps [26] (1977), has been widely exploited in literature. In a REE perspective, investors are assumed to formulate rational beliefs on the occurring events in the market and their behavior is driven by the principle of utility maximization. This should provide theorical support to the efficient market hypothesis (EMH), proposed by Fama [11] (1965), which is one of the most influential and controversial topic in modern finance. EMH assumes that equilibrium stock prices in real financial markets are necessarily informationally efficient, in the sense that equilibrium prices embody the current true values of the stocks. More specifically, EMH suggests equilibrium stock prices arise as slightly deviations of the expected present value of the cumulative future stock dividend yields. The extent of such deviations gives rise to the classification of financial market in strong form efficient, semi-strong form efficient and weak form efficient. According to our knowledge, the exploitation of a REE perspective to financial market models has so far led to equilibrium stock prices which verify the strong or the semi-strong form of EMH. Nevertheless, despite Jensens' statement: "there is no other proposition in Economics which has more solid empirical evidence supporting it than EMH" (see Jensen [25] (1978)), an increasing number of empirical studies advocating EMH have lately been challenged and even reversed. In a EMH setting, some relevant phenomena, commonly known as "market anomalies", such as the equity premium puzzle (see Mehra & Prescott [31] (1985)), the excess volatility in stock returns and price-dividend ratios (see Grossman & Shiller [20] (1981), LeRoy & Porter [28] (1981), Shiller [37] (1981)), the predictability of stock returns (see Poterba & Summers [35] (1988), Fama & French [12] (1989), see also Campbell & Shiller [5] (1988)), cannot be explained. The doubts on the validity of EMH have consequently affected also REE models and have led many authors to a sceptical attitude towards their validity. Even a new approach to Finance, known as "Behavioral Finance" (see Shleifer [38] (2000)), which relaxes both the assumptions of individual rationality and consistent beliefs, has been developed to show how the trading activity of boundedly rational investors may significantly deviate the prices of the risky assets from their fundamental values. This would propose possible interpretations of the market anomalies. (see e.g. DeLong & als [8] (1990), Benartzi & Thaler [3] (1995), Timmermann [41] (1996)). However, our results suggest that, still in a rational expectation setting, in which operate risk averse investors who formulate consistent beliefs and maximize their utility, the increasing of the rational investors' risk aversion or market noise volatility leads them to achieve a higher utility in market equilibria characterized by prices exhibiting even a large extent of informational inefficiency. Hence, our models seem to offer a possible interpretative key of the several inefficiencies observed in stock prices of real financial markets without abandoning the REE paradigm.

In Wang's paper [42] (1993), two groups of asymmetrically informed risky averse rational investors trade in a frictionless incomplete market with an infinite horizon, exchanging continuously in time a risk free asset for a stock and counsuming a single commodity. The risk free asset rewards with a constant rate of return, while the stock pays a continuous flow of dividends growing at a stochastic rate. Public information consists of history of the realizations of the stock price and the dividend yields, as well as the values of all exogenous parameters of the model. In an imperfect competitive rational expectation equilibrium perspective, the investors of both groups determine their optimal demand schedule for the stock, in terms of a putative equilibrium price, and submit it to an implicit Walrasian auctioneer. The latter aggregates the investors' demand schedules and sets the actual equilibrium price via the market clearing condition, on account of the current supply of the stock. Incompleteness of the market is modeled by introducing a continuous stochastic shock on the total supply of the stock, as a possible effect of the trading activity of non rational investors. Asymmetric information is realized by providing the rational investors of one group with a private information on the growth rate of the dividend yield of the stock. This allows such privately informed investors to have a sharper knowledge of the future growth rate of dividends and to anticipate more accurate expected returns from investing in the stock. Eventually, the privately informed investors can also observe the history of the realizations of the shock on the stock supply, thereby ending up with complete information. The rational investors of the other group can directly observe neither the realizations of the private information signal nor the realizations of the stochastic shock on the stock supply. On the other hand, since the growth rate of dividends determines the rate of appreciation of stock prices, changes of prices provide signals about the future growth of dividends. Hence, such uninformed investors will rationally extract information about the state of the economy from prices as well as dividends. However, , due to the market incompleteness, the observed signals do not fully reveal the true values of all the state variables of the economy. In Wang's asymmetric information setting, stationary equilibria are possible thanks to the market incompleteness, which prevents the intervention of the No-Trade Theorem (see Grossman & Stiglitz [21] (1980)). However, Wang's putative equilibrium stock price deviates from the stock fundamental value [1], by a discount term accounting for investors' risk aversion, a term expressing a linear sensitivity of the price to the supply shocks, and an additional term modeling a linear response to the information asymmetry. This is a direct generalization, accounting for investors' information asymmetry, of the putative equilibrium stock price proposed by Campbell & Kyle [4] (1993). As a consequence, Wang's actual equilibrium price is ex-ante informationally efficient in the semi-strong form[2]. On the other hand, such a putative equilibrium stock price introduces an ex-ante constraint on the extent of imperfect competition embodied by the informed and uninformed investors' trading strategies. In particular, in

[1] i.e. the expected present value of the cumulative future dividend payments under complete information.

[2] The strong form information efficiency is achieved in the long run limit, because the private information is revealed over time.

the only equilibrium candidate of the model revealed by Wang[3], the uninformed investors end up with exploiting only their estimate on the noisy component of the market demand, while the uninformed investors end up with exploiting only their knowledge of the shock on the stock supply and the estimation errors on their private information made by the uninformed investors. In fact, the structure of Wang's equilibrium stock price necessarily implies that the optimal demands for the stock of the two groups of investors have null correlation with both the variables of the model conveying the public and private information. In addition, in Wang's equilibrium candidate, the uninformed investors' demands are positively correlated with their estimate of the stock supply shocks, while the informed investors' demands are positively correlated with both the stock supply shocks and the uninformed investors' estimation errors of the private information[4].

The first difference between Wang's model and our models is that we don't assume the putative equilibrium stock price as a deviation from the stock fundamental value. Indeed, we wonder why the equilibrium stock price should be set as ex-ante informationally efficient in the semi-strong form in a model where the aggregation of the market demand is due to a Walrasian auctioneer instead of market makers. In fact, Walrasian auctioneer's goal is only to aggregate the investors' demand schedules for the stock and set the equilibrium price, according to the market clearing condition. Hence, we can see no reason for her to not set an inefficient price, provided this yields a higher utility to the investors than an efficient one. Second, in our models the shock on the supply stock exihibits a correlation with the shock on the dividend flow. This because we aim to model the demand of non rational traders who can overreact or underreact to dividend surprises, rather than being driven by purely liquidity needs. Third, in our second model, following Guo & Kyle [22] (2008), we aim to analyze the effect produced on the equilibrium stock price of the introduction in the market of transaction costs.

Our approach allows us to discover many market equilibrium candidates in which the prices no longer need to be a linear perturbation of the stock fundamental value and the investors' optimal demands may have non null

[3]Still in the same Wang's setting other equilibrium candidates are possible, in which the rational investors' trading stategies exihibit a larger extent of imperfect competition (see Monte, Perrotta, Fabretti, [32] (2010)).

[4]The informed investors, thanks to their complete information, are in a position to detect the uninformed investors' estimate of the private information, while the uninformed investors' estimate of the private information is equivalent to their estimate of the risky asset supply shocks.

correlation with some information source. Otherwise saying, the investors transmit information to the maket while exploiting their optimal demands. This leads us to refer to this type of equilibrium candidates as *strategic*. In addition, we have discovered multiple equilibrium candidates of Wang's type itself.

More specifically, we have found that, while Wang's equilibrium candidate of the model is revealed also via our approach[5], there exist other equilibrium candidates which still could be revealed by Wang's approach and strategic candidates which cannot be obtained via Wang's approach. This is not that much surprising though, because our seek of equilibrium candidate leads to a relaxation of the constraints on the coefficients of Wang's conjectured equilibrium risky asset price, making the latter a linear perturbation of its fundamental value. However, some of the strategic candidates are clearly to be rejected for their lack of Pareto efficiency, since they are characterized by a lower expected utility for both the groups of investors than the benchmark of Wang's candidate. On the other hand, in some other candidates the expected utility reduces for a group of investors while increases for the other group, which doesn't lead to a Pareto rejection, or even increases for both the groups. Therefore, our main result is the discovery that investors' risk aversion or stock supply volatility are crucial in determining the actual equilibrium of the model. In fact, while under low investors' risk aversion and low stock supply volatility we have revealed no strategic equilibrium candidates yielding a higher expected utility to both the groups of investors than Wang's benchmark, under high investors' risk aversion or high stock supply volatility equilibrium candidates have been revealed in which the investors of both the groups increase their expected utility. The stock prices characterizing this strategic equilibrium candidates generally yield a much stronger discount for holding the stock and have a much stronger sensitivity to the supply shocks than the corresponding Wang's price. In addition, they exhibit some extent of informational inefficiency. Our interpretation of these results is that a high perception of market risk, expressed in terms of investors' risk aversion and noise market volatility, leads risk-averse investors to stress the strategic features of their trading, with lack of efficiency in equilibrium stock prices.

Our main computational procedure[6] to achieve the equilibrium candi-

[5] We stress once more that via our approach, Wang's equilibrium is revealed ex post since it is anticipated in no assumption on the coefficients of the putative equilibrium price of the model.

[6] All computational procedures of ours have been implemented in Wolfram Mathematica® environment

dates consists of a sequential bargaining procedure. In this procedure each group of investors progressively adjust their demand for the stock, as an optimal response to the demand of the other group. The stock price moves accordingly. The equilibrium is declared when no further significant adjustments occur.

The thesis is organized as follows. In Chapter *Mathematical Tools* we describe the most important results concerning the mathematical tools essential to our financial models. In Chapter *Asymmetric Information* we describe the first model and in its Subsections *Uninformed Investors' Filtering Optimization Problem* and *Informed Investors' Optimization problem* we present our approach to the uniformed and informed investors' optimization problem, respectively. In Subsection *Degenerate Cases* we describe the cases in which all investors are informed, or uninformed, respectively; these cases are useful for the simulation of the model. In Chapter *Model Simulation by Computational Procedures* we give a characterization of the Bayesian-Nash equilibrium, explain our computational procedures, establish formulas to compare Monte-Perrotta-Fabretti and our approach with Wang's one. After we presents some achieved results. In Chapter *Asymmetric Information with Transaction Costs* we expose our second and theoretical model which combines the transaction costs of the previous model. Chapter *Conclusions* concludes. In Appendix *Square Root Matrix* we explain the way by which we have obtained explicitly the square root of a matrix, in Appendix *Proof of Propositions* we show the proofs of transaction costs' model, and eventually, in Appendix *Noise Trader's Wealth* and in Appendix *Traders' Wealth Equation*, we show as stock supply's equation and wealth's equation are obtained, respectively.

In this summary we choose to expose only the second model. Indeed, while the first model analyzed is an extension of Wang's model and (see also Monte, Perrotta & Fabretti in [32] (2010)), by introducing transaction costs in rational investors' wealth equations, we have develop a fresh model, not yet presented in literature, which is significantly different, both in conceptual and numerical terms, from Wang's model and its extensions.

Chapter 1

Asymmetric Information with Transaction Costs

1.1 The Model

We consider an infinite-horizon economy with a single commodity, where a risk free asset and a risky asset, hereinafter *stock*, are traded continuously in time, in a market with friction. The risk free asset rewards with a constant rate of return $r > 0$ while the stock yields a continuous dividend rate $D(t)$ whose dynamics is given by the equation

$$dD(t) = (\Pi(t) - \alpha_D D(t))dt + \sigma_{D,D}\, dw_D(t) + \sigma_{D,\Pi}\, dw_\Pi(t). \tag{1.1}$$

This characterizes a mean reverting process towards the stochastic level $\alpha_D^{-1}\Pi(t)$, driven in turn by the null mean reverting process

$$d\Pi(t) = -\alpha_\Pi \Pi(t)dt + \sigma_{\Pi,\Pi}\, dw_\Pi(t). \tag{1.2}$$

In (1.1) and (1.2), the terms $w_D(t)$ and $w_\Pi(t)$ are independent standard Wiener processes, the positive parameter α_D [resp. α_Π] is the constant mean speed of reversion of the process $D(t)$ [resp. $\Pi(t)$] around its long-run level, the differential $\sigma_{D,D} dw_D(t) + \sigma_{D,\Pi} dw_\Pi(t)$ [resp. $\sigma_{\Pi,\Pi} dw_\Pi$], for constant $\sigma_{D,D}$, $\sigma_{D,\Pi}$ [resp. $\sigma_{\Pi,\Pi}$], constitutes the innovation in $D(t)$ [resp. $\Pi(t)$], and the quantity $\sigma^2_{D,D} + \sigma^2_{D,\Pi} \equiv \sigma^2_D$ [resp. $\sigma^2_{\Pi,\Pi}$] is the innovation variance of $D(t)$ [resp. $\Pi(t)$]. The choice of a positive [resp. negative] $\sigma_{D,\Pi}$ causes a positive [resp. negative] correlation between changes in dividend rate $D(t)$ and the signal $\Pi(t)$. Setting $\sigma_{D,\Pi} = 0$ makes independent innovations in $D(t)$ and $\Pi(t)$.

The interpretation of $\Pi(t)$ as a private information on $D(t)$ follows by remarking that the quantity

$$\mathbf{E}[\Pi(t)] - \alpha_D D(t) \qquad [\text{resp.} \quad \Pi(t) - \alpha_D D(t)]$$

approximates the growth rate of dividend rate process at time $t + \Delta t$, which is expected by an investor whose information up to t is restricted to the only history of dividend rate process itself [resp. includes both the histories of dividend rate and the informative signal].

Similar to Wang [42] (1993), we also assume that the total supply of the stock in the market is stochastic with a stationary level normalized to 1. However, with the goal of dealing with transaction costs, we model the current deviation of the stock supply from its long-run stationary level by means of a "smooth" process $\Upsilon(t)$ fulfilling the differential equation

$$d\Upsilon(t) = \Theta(t)\, dt \tag{1.3}$$

where $\Theta(t)$ is a null mean reverting process driven by the equation

$$d\Theta(t) = -\alpha_\Theta \Theta(t)dt + \sigma_{\Theta,D} dw_D(t) + \sigma_{\Theta,\Theta}\, dw_\Theta(t). \tag{1.4}$$

In (1.4), $w_\Theta(t)$ is a standard Wiener process, which is independent of $w_D(t)$ and $w_\Pi(t)$, the positive parameter α_Θ gives the constant mean speed of reversion of the processes $\Theta(t)$ towards its null long-run level, the parameter $\sigma_{\Theta,D}$ expresses the correlation between $\Theta(t)$ and $D(t)$, the differential $\sigma_{\Theta,D} dw_\Theta(t) + \sigma_{\Theta,\Theta}\, dw_\Theta(t)$ is the innovation in $\Theta(t)$ and the quantity $\sigma^2_{\Theta,D} + \sigma^2_{\Theta,\Theta} \equiv \sigma^2_{\Theta,\Theta}$ is the innovation variance of $\Theta(t)$.

The stochastic supply of the stock can be equivalently interpreted in terms of the presence in the market of non rational investors, whose demand for the stock $\Upsilon(t)$, introduces a "smoothly" changing noise component in the aggregate market demand. Accordingly, $\Theta(t)$ should be interpreted as the non rational investors' order flow. In light of this, the choice of a positive $\sigma_{\Theta,D}$ causes a negative correlation between changes in dividend rate $D(t)$ and changes in the noisy component of the market order flow: when positive [resp. negative] changes occur in the dividend, the noisy component of the market order flow is affected by a positive [resp. negative] contribute. This models an overreaction of the non rational investors to dividend surprises. By contrast, the choice of a negative $\sigma_{\Theta,D}$ represents an underreaction of the non rational investors to dividend surprises. Setting $\sigma_{\Theta,D} = 0$ we model non rational investors trading only for liquidity reason with dividend-inelastic demand.

All the rational investors who participate in the market know the market structure and the publicly available history of the realizations of the stock price and dividend, up to the current time t. However, only some of them can observe the history of the realizations of the private information signal $\Pi(t)$, up to t. In addition, we assume that these privately informed investors can also observe the realizations of the non rational investors' order flow $\Theta(t)$ (see Wang [42, Footnote 14 p. 253] (1993)). Eventually, the observation of $\Theta(t)$ can be justified by the assumption that the privately informed investors trade on the floor of the financial exchange (see Back cite [1]). This asymmetric information setting leads to split the rational investors in two groups, which we denote by the labels I and U, according to whether they are privately informed or not. All investors of each group are endowed with constant absolute risk aversion and same preferences. Therefore, it is possible to deal with them as they are a single representative rational agent whose inventory, namely the holding of the stock, aggregates the inventories of all investors belonging to the group. We denote by $\Psi_K(t)$ the inventory of the K representative agent at time t, for $K = I$, U. Hence, in equilibrium, the total supply of the stock satisfies the market clearing condition

$$(1-\omega)\Psi_I(t) + \omega\Psi_U(t) = 1 - \Upsilon(t), \tag{1.5}$$

where $\omega \in [0,1]$ is a parameter modeling the fraction of the uninformed investors in the market: when $\omega = 0$ [resp. $\omega = 1$] the rational investors are all privately informed [resp. uninformed]. Similarly to the non rational investors, the representative agents are assumed to trade "smoothly". More specifically, writing $\Phi_K(t)$ for the K representative rational agent's order flow, we assume

$$d\Psi_K(t) = \Phi_K(t)dt, \qquad K = I, U. \tag{1.6}$$

In addition, following Guo & Kyle [22] (2008) (see also Garleanu & Pedersen [15] (2009)) we assume the representative rational agents incurr in a quadratic instantaneous transation cost of the form

$$\frac{1}{2}\kappa\Phi_K^2(t), \qquad K = I, U \tag{1.7}$$

where $\kappa \geq 0$. The latter means a transation cost per unit of time proportional to the order flow times the speed of the order flow, that is

$$\frac{1}{2}\kappa\Phi_K^2(t)dt = \frac{1}{2}\kappa\frac{d\Psi_K(t)}{dt}\cdot d\Psi_K(t) = \frac{1}{2}\kappa\Phi_K(t)\cdot d\Psi_K(t), \qquad K = I, U. \tag{1.8}$$

As a consequence, the dynamics of the investors's wealth, $W_K(t)$, turns out to be a solution to the stochastic differential equation

$$dW_K(t) = (rW_K(t) - c_K(t) - \frac{1}{2}\kappa\Phi_K^2(t))\,dt + \Psi_K(t)dQ(t), \qquad K = I, U, \tag{1.9}$$

where $c_K(t)$ is the K representative agent's consumption rate and $Q(t)$ is the instantaneous excess return to one share of stock fulfilling

$$dQ(t) \equiv (D(t) - rP(t))\,dt + dP(t), \tag{1.10}$$

given the stock price $P(t)$.

In real markets transation costs are usually constant or proportional to the order flow[1]. However, as we will show in the sequel, the assumption of quadratic transation costs is a device that allows smooth equilibria of the model characterized by an informative order flow, while capturing the cumulative effect of several type of frictions in the market.

Both the representative agents maximize the expected value of the discounted utility rate of their consumption over the infinite time-horizon by controlling their order flow $\Phi_K(t)$ and their consumption rate $c_K(t)$. Formally, the K representative agent's objective function can be written as

$$\max_{\Phi_K(\cdot), c_K(\cdot)} \left\{ \mathbf{E}\left[\int_t^{+\infty} -e^{-(\rho_k s + \varphi_k c_k(s))}\,ds | \mathfrak{F}_K(t) \right] \right\}, \qquad K = I, U, \tag{1.11}$$

where ρ_K is the K representative agent's subjective rate of time preference, φ_K is the coefficient of her absolute risk aversion, $\mathfrak{F}_K(t)$ stands for the σ-field representing the K representative agent's information up to the current instant t, and $\mathbf{E}[\,\cdot\,|\mathfrak{F}_K(t)]$ is the conditional expectation operator given $\mathfrak{F}_K(t)$. In turn, $\Phi_K(t)$ and $c_K(t)$ are subject to the dynamics of the K representative agent's wealth (1.9) and the other variables of the economy.

The idea of rational agents maximizing the expected value of the discounted utility rate of their consumption by controlling their order flows rather than their inventories is a crucial, but not new, characteristic of the model under investigation (see Gennotte & Kyle [16] (1991), Back [1] (1992), see also Guo & Kyle [22] (2008)). Actually. it is the key feature which allows us to introduce transation costs within Wang's model.

As above mentioned, both the representative agents can observe the history of realizations of the dividend $D(t)$ and the price $P(t)$ of the stock,

[1] In particular, investors who prefer to trade large [small] amounts of stocks usually incurr in constant [proportional] transation cost.

which are public information. In addition the informed representative agent can observe also the history of the realizations of the signals $\Pi(t)$ and $\Theta(t)$. Therefore, since we analyze the model under the assumption of perfect initial information, the informed investors can clearly observe also $\Upsilon(t)$. Hence, thanks to the market clearing condition (1.5) and its differential form

$$(1-\omega)\Phi_I(t) + \omega\Phi_U(t) = -\Theta(t), \tag{1.12}$$

the knowledge of $\Upsilon(t)$ and $\Theta(t)$ allows the informed representative agent to know also $\Psi_U(t)$ and $\Phi_U(t)$. Ultimately, the informed representative agent holds complete information, strictly superior to the uninformed representative agent's information, who can observe neither $\Pi(t)$ nor $\Theta(t)$. However, in equilibrium, still by virtue of (1.5) and (1.12), the uninformed representative agent can observe the part of the stock supply

$$T(t) \equiv 1 - \Upsilon(t) - \omega\Psi_I(t) \tag{1.13}$$

and the stock order flow

$$U(t) = -\Theta(t) - \omega\Phi_I(t) \tag{1.14}$$

which are complementary with respect to her own inventory $\Psi_U(t)$ and order flow $\Phi_U(t)$, respectively. She cannot disaggregate the observed $T(t)$ and $U(t)$, though.

We then have

$$\mathfrak{F}_U(t) = \sigma(P(s), D(s), \Pi(s), \Theta(s), \Upsilon(s), \Phi_U(s), \Psi_U(s);\ s \leq t), \tag{1.15}$$

and

$$\mathfrak{F}_U(t) = \sigma(P(s), D(s), T(s), U(s);\ s \leq t). \tag{1.16}$$

1.1.1 Equilibrium

To define a linear equilibrium of the model, we partially follow Wang [42] (1993). Similar to Wang, we assume that each representative agent conjectures a putative equilibrium stock price which is linear in all variables of the economy conveying information to herself or her competitor. Accordingly, the agent determines her corresponding optimal order flow schedule for the stock, which is transmitted to a Walrasian auctioneer. The latter aggregates the representative agents' optimal order flow schedules and sets the actual equilibrium stock price via the market clearing condition. However, differently than Wang, we don't assume that the putative equilibrium stock price characterizes ex-ante as a linear perturbation of the stock fundamental value[2] by a linear combination of the other variables of the economy. Indeed, we are interested in finding conditions on the exogenous parameters of the economy leading to prices with different extents of informational efficiency. Moreover, in our model, to determine her optimal order flow schedule, the uninformed representative rational agent needs to forecast the private information held by her competitor. This task can be rationally accomplished by a Kalman-Bucy linear forecast. Therefore, the equilibrium is achieved not only via the market clearing condition, but also via the confirmation of the uninformed representative agent's rational forecast of her informed competitor's optimal behavior. Hence, we are in front of a Bayesian-Nash (imperfect competitive) equilibrium.

More specifically, both the agents conjecture a stock price in the form

$$\begin{aligned} P(t) = p_1 + p_D D(t) + p_\Pi \Pi(t) + p_\Theta \Theta(t) + p_\Upsilon \Upsilon(t) + p_{\Psi_I} \Psi_I(t) + p_{\Psi_U} \Psi_U(t) \\ + p_{\hat{D}} \hat{D}(t) + p_{\hat{\Pi}} \hat{\Pi}(t) + p_{\hat{\Theta}} \hat{\Theta}(t) + p_{\hat{\Upsilon}} \hat{\Upsilon}(t) + p_{\hat{\Psi}_I} \hat{\Psi}_I(t) + p_{\hat{\Psi}_U} \hat{\Psi}_U(t), \end{aligned} \tag{1.17}$$

for misspecified constant coefficients p_1, p_D, p_Π, $\ldots$, $p_{\hat{\Psi}_U}$, where $\hat{J}(t) \equiv \mathbf{E}[J(t)|\mathfrak{F}_U(t)]$, for $J = D, \Pi, \Theta, \Upsilon, \Psi_I, \Psi_U$, and the estimates $\hat{D}(t)$ and $\hat{\Psi}_U(t)$ of the uninformed representative agent's observed variables $D(t)$ and $\Psi_U(t)$ are introduced only for notational symmetry. Actually, with no loss in the generality, we can set $p_{\hat{D}} = p_{\hat{\Psi}_U} \equiv 0$.

Now, to tackle her optimization problem (1.11) via the standard Bellman approach, the uninformed representative agent aims to rewrite $P(t)$ only in terms of the variables she can directly observe and the rational estimates of the variables she cannot directly observe. In addition, the innovation in the dynamics of $P(t)$ has to be expressed in terms of a multidimensional Wiener process, which generates exactly her information $\mathfrak{F}_U(t)$. This because she needs to deal with a Markov process. Thereafter, the Separation Principle (see e.g. Fleming & Rishel [13] (1975)) can be exploited.

Clearly, from the uninformed representative agent's point of view, the publicly observed stock price can be rewritten as

$$\begin{aligned} P(t) = p_1 + p_D D(t) + (p_\Pi + p_{\hat{\Pi}}) \hat{\Pi}(t) + (p_\Theta + p_{\hat{\Theta}}) \hat{\Theta}(t) \\ + (p_\Upsilon + p_{\hat{\Upsilon}}) \hat{\Upsilon}(t) + (p_{\Psi_I} + p_{\hat{\Psi}_I}) \hat{\Psi}_I(t) + p_{\Psi_U} \Psi_U(t). \end{aligned} \tag{1.18}$$

[2] That is the expected present value of the cumulative dividend payments under complete information.

Nevertheless, the equations driving the dynamics of the uninformed representative agent's estimates $\hat{\Pi}(t)$, $\hat{\Theta}(t)$, $\hat{\Upsilon}(t)$, and $\hat{\Psi}_I(t)$ still need to be determined. Now, setting $y_U(t) \equiv (D(t), \Pi(t), \Theta(t), \Upsilon(t), \Phi_I(t), \Psi_I(t))^\intercal$ the equation for $\hat{y}_U(t) \equiv \mathbf{E}[y_U(t)|\mathfrak{F}_U(t)] \equiv (\hat{D}(t), \hat{\Pi}(t), \hat{\Theta}(t), \hat{\Upsilon}(t), \hat{\Phi}_I(t), \hat{\Psi}_I(t))^\intercal$ can be obtained by a Kalman Bucy filtering procedure, provided the equation for $\Phi_I(t)$ is known. This leads the uninformed representative agent to conjecture the optimal informed agent's order flow schedule $\Phi_I(t)$. Therefore, following Gennotte & Kyle [16] (1991), we assume that the uninformed representative agent conjectures the optimal informed agent's order flow schedule in the form

$$\begin{aligned}\Phi_I(t) = {} & \phi_{I,D}D(t) + \phi_{I,\Pi}\Pi(t) + \phi_{I,\Theta}\Theta(t) + \phi_{I,\Upsilon}\Upsilon(t) + \phi_{I,\Psi_I}\Psi_I(t) \\ & + \phi_{I,\Phi_U}\Phi_U(t) + \phi_{I,\Psi_U}\Psi_U(t) + \phi_{I,\hat{D}}\hat{D}(t) + \phi_{I,\hat{\Pi}}\hat{\Pi}(t) + \phi_{I,\hat{\Theta}}\hat{\Theta}(t) + \\ & \phi_{I,\hat{\Upsilon}}\hat{\Upsilon}(t) + \phi_{I,\hat{\Psi}_I}\hat{\Psi}_I(t) + \phi_{I,\hat{\Phi}_I}\hat{\Phi}_I(t) + \phi_{I,\hat{\Phi}_U}\hat{\Phi}_U(t) + \phi_{I,\hat{\Psi}_U}\hat{\Psi}_U(t).\end{aligned} \tag{1.19}$$

Indeed, the uninformed representative agent knows that her rational competitor is a position to replicate her estimates since the latter is completely informed. Hence, in a imperfect competitive perspective, it is natural to assume that the uninformed representative agent conjectures that her informed competitor fully exploits all the variables conveying information and her estimates of these variables. On the other hand, in equilibrium, the market clearing conditions (1.5) and (1.12) make the variables $\Psi_U(t)$, $\Phi_U(t)$ redundant and so are the variables $\hat{\Phi}_U(t)$ and $\hat{\Psi}_U(t)$, on account of the equalities $\hat{\Psi}_U(t) = \Psi_U(t)$ and $\hat{\Phi}_U(t) = \Phi_U(t)$. Moreover, also the variable $\hat{D}(t) = D(t)$ is redundant. Thus, with no loss in the generality, we can set

$$\phi_{I,\Phi_U} = \phi_{I,\Psi_U} = \phi_{I,\hat{D}} = \phi_{I,\hat{\Phi}_U} = \phi_{I,\hat{\Psi}_U} = 0.$$

Hence, the conjectured $\Phi_I(t)$ can then rewritten as

$$\begin{aligned}\Phi_I(t) = {} & \phi_{I,D}D(t) + \phi_{I,\Pi}\Pi(t) + \phi_{I,\Theta}\Theta(t) + \phi_{I,\Upsilon}\Upsilon(t) + \phi_{I,\Psi_I}\Psi_I(t) \\ & + \phi_{I,\hat{\Pi}}\hat{\Pi}(t) + \phi_{I,\hat{\Theta}}\hat{\Theta}(t) + \phi_{I,\hat{\Upsilon}}\hat{\Upsilon}(t) + \phi_{I,\hat{\Phi}_I}\hat{\Phi}_I(t) + \phi_{I,\hat{\Psi}_I}\hat{\Psi}_I(t) \\ & = \phi_I^\intercal y_U(t) + \hat{\phi}_I^\intercal \hat{y}_U(t),\end{aligned} \tag{1.20}$$

where

$$\phi_I^\intercal \equiv (\phi_{I,D}, \phi_{I,\Pi}, \phi_{I,\Theta}, \phi_{I,\Upsilon}, 0, \phi_{I,\Psi_I}), \quad \hat{\phi}_I^\intercal \equiv (0, \phi_{I,\hat{\Pi}}, \phi_{I,\hat{\Theta}}, \phi_{I,\hat{\Upsilon}}, \phi_{I,\hat{\Phi}_I}, \phi_{I,\hat{\Psi}_I}). \tag{1.21}$$

This clearly implies

$$d\Phi_I(t) = \phi_I^\intercal dy_U(t) + \hat{\phi}_I^\intercal d\hat{y}_U(t). \tag{1.22}$$

As we will show in Section 1.2, Equation (1.22) allows the uninformed representative agent to perform her filtering-optimization procedure.

On the other hand, the form (1.17) of the conjectured price, allows the informed representative agent to directly perform her optimization procedure. In fact, in equilibrium, the informed representative agent observes all the variables appearing in (1.17), knows the equations for $D(t)$, $\Pi(t)$, $\Theta(t)$, and $\Upsilon(t)$, is in a position to replicate the equation for the estimates $\hat{\Pi}(t)$, $\hat{\Theta}(t)$, $\hat{\Upsilon}(t)$, and $\hat{\Psi}_I(t)$, considers the equation for $\Psi_I(t)$ in terms of her control $\Phi_I(t)$ and, by virtue of (1.12), can consider the equation

$$d\Psi_U(t) = -\frac{1}{\omega}\left((1-\omega)\Phi_I(t) + \Theta(t)\right). \tag{1.23}$$

1.2 Uninformed Representative Agent's Filtering Optimization

As a consequence of the above considerations, the uninformed representative agent can write the state equation

$$dy_U(t) = A_U y_U(t)dt + Q_U^{\dagger}\, dw(t) + e_{\Phi_I}\, d\Phi_I(t), \tag{1.24}$$

where

$$A_U \equiv \begin{pmatrix} -\alpha_D & 1 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_\Pi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_\Theta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix},$$

$$Q_U^{\dagger} \equiv \begin{pmatrix} \sigma_{D,D} & \sigma_{D,\Pi} & 0 \\ 0 & \sigma_{\Pi,\Pi} & 0 \\ \sigma_{\Theta,D} & 0 & \sigma_{\Theta,\Theta} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e_{\Phi_I} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

and $w(t) \equiv (w_D(t), w_\Pi(t), w_\Theta(t))^{\intercal}$. Combining (1.24) with (1.22), it then follows

$$dy_U(t) = A_U y_U(t)\, dt + Q_U^{\dagger}\, dw(t) + e_{\Phi_I}\phi_I^{\intercal}\, dy_U(t) + e_{\Phi_I}\hat{\phi}_I^{\intercal}\, d\hat{y}_U(t),$$

which implies

$$(I - e_{\Phi_I}\phi_I^\intercal)dy_U(t) = A_U y_U(t)dt + e_{\Phi_I}\hat{\phi}_I^\intercal\, d\hat{y}_U(t) + Q_U^\dagger\, dw(t). \qquad (1.25)$$

On the other hand, a direct check shows that the matrix $I - e_{\Phi_I}\phi_I^\intercal$ is invertible with inverse $(I - e_{\Phi_I}\phi_I^\intercal)^{-1} = I + e_{\Phi_I}\phi_I^\intercal$[3]. Therefore, it is possible to write

$$dy_U(t) = A_{U,\Phi} y_U(t)\, dt + B_{U,\Phi}\, d\hat{y}_U(t) + Q_{U,\Phi}^\dagger\, dw(t), \qquad (1.26)$$

where

$$A_{U,\Phi} \equiv (I+e_{\Phi_I}\phi_I^\intercal)A_U, \quad B_{U,\Phi} \equiv (I+e_{\Phi_I}\phi_I^\intercal)e_{\Phi_I}\hat{\phi}_I^\intercal, \quad Q_{U,\Phi}^\dagger \equiv (I+e_{\Phi_I}\phi_I^\intercal)Q_U^\dagger. \qquad (1.27)$$

To find the equations of the estimates $\hat{y}_U(t)$, the uninformed representative agent needs to write the equations of the observed signals. As discussed above, besides the information provided by the history of the dividend rate $D(t)$, the uninformed investor observes the signal $T(t)$ from the (1.12), and, from the publicy observable stock price $P(t)$, the signal

$$S(t) = p_\Pi \Pi(t) + p_\Theta \Theta(t) + p_\Upsilon \Upsilon(t) + p_{\Psi_I}\Psi_I(t). \qquad (1.28)$$

Thus, introducing the uninformed representative agent's observation vector $y_{U,O}(t) \equiv (D(t), S(t), U(t))^\intercal$, such that $\mathfrak{F}_U(t) = \sigma(y_{U,O}(t))$, we have

$$dy_{U,O}(t) = M\, dy_U(t), \qquad (1.29)$$

where

$$M \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_\Pi & p_\Theta & p_\Upsilon & 0 & p_{\Psi_I} \\ 0 & 0 & -1 & 0 & -\omega & 0 \end{pmatrix}. \qquad (1.30)$$

[3] In fact, since

$$e_{\Phi_I}\phi_I^\intercal e_{\Phi_I}\phi_I^\intercal = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_{I,D} & \phi_{I,\Pi} & \phi_{I,\Theta} & \phi_{I,\Upsilon} & 0 & \phi_{I,\Psi_I} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_{I,D} & \phi_{I,\Pi} & \phi_{I,\Theta} & \phi_{I,\Upsilon} & 0 & \phi_{I,\Psi_I} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = 0,$$

we have

$$(I - e_{\Phi_I}\phi_I^\intercal)(I + e_{\Phi_I}\phi_I^\intercal) = I - e_{\Phi_I}\phi_I^\intercal + e_{\Phi_I}\phi_I^\intercal + e_{\Phi_I}\phi_I^\intercal e_{\Phi_I}\phi_I^\intercal = I = (I + e_{\Phi_I}\phi_I^\intercal)(I - e_{\Phi_I}\phi_I^\intercal)$$

Therefore, combining (1.29) with (1.26), it follows that the equation driving the dynamics of the observation vector is given by

$$dy_{U,O}(t) = A_{U,O}y(t)\,dt + B_{U,O}\,d\hat{y}(t) + Q^{\dagger}_{U,O}\,dw(t), \tag{1.31}$$

where

$$A_{U,O} \equiv MA_{U,\Phi}, \qquad B_{U,O} \equiv MB_{U,\Phi}, \qquad Q^{\dagger}_{U,O} \equiv MQ^{\dagger}_{U,\Phi}. \tag{1.32}$$

Hence, the uninformed representative agent's filtering problem becomes the determination of the equation for $\hat{y}_U(t)$, given the state equation (1.26) and the observed signal equation (1.29). On the other hand, the estimate vector $\hat{y}_U(t)$ enters Equation (1.26). This does not allow a straightforward application of the standard Kalman-Bucy linear filtering procedure to deduce the equation for $\hat{y}_U(t)$. However, we are interested in determining the linear equilibria of the model. Therefore, guided by the structure of the estimate dynamics in the standard Kalman-Bucy setting and following Gennotte & Kyle [16], with no loss in the generality we assume the uninformed representative agent aims to update her estimates according to the equation

$$d\hat{y}_U(t) = G(t)\hat{y}_U(t)dt + H(t)dy_{U,O}(t), \tag{1.33}$$

for suitable constant matrices $H(t) \equiv (h_{j,k}(t))_{j,k=1,\ldots,6}$ and $G(t) \equiv (g_{j,k}(t))_{j,k=1,\ldots,6}$. Now, combining (1.33) with (1.31), it follows

$$d\hat{y}_U(t) = H(t)A_{U,O}y_U(t)\,dt + G(t)\hat{y}_U(t)\,dt + H(t)B_{U,O}\,d\hat{y}_U(t) + H(t)Q^{\dagger}_{U,O}\,dw(t),$$

that is

$$d\hat{y}_U(t) = A_{U,H}(t)y_U(t)\,dt + B_{U,H}(t)\hat{y}_U(t)\,dt + Q^{\dagger}_{U,H}(t)\,dw(t), \tag{1.34}$$

where

$$\begin{aligned} A_{U,H}(t) &\equiv (I - H(t)B_{U,O})^{-1}H(t)A_{U,O}, \\ B_{U,H}(t) &\equiv (I - H(t)B_{U,O})^{-1}G(t), \\ Q^{\dagger}_{U,H}(t) &\equiv (I - H(t)B_{U,O})^{-1}H(t)Q^{\dagger}_{U,O}. \end{aligned} \tag{1.35}$$

under the assumption that $(I - H(t)B_{U,O})^{-1}$ is invertible.

Finally, replacing 1.34) into (1.26) and (1.31), we obtain

$$dy_U(t) = A_{U,\Phi,H}(t)y_U(t)\,dt + B_{U,\Phi,H}(t)\hat{y}_U(t)\,dt + Q^{\dagger}_{U,\Phi,H}(t)\,dw(t), \tag{1.36}$$

where

$$A_{U,\Phi,H}(t) \equiv A_{U,\Phi} + B_{U,\Phi} A_{U,H}(t), \tag{1.37}$$
$$B_{U,\Phi,H}(t) \equiv B_{U,\Phi} B_{U,H}(t),$$
$$Q^{\dagger}_{U,\Phi,H}(t) \equiv B_{U,\Phi} Q^{\dagger}_{U,H}(t) + Q^{\dagger}_{U,\Phi},$$

and

$$dy_{U,O}(t) = A_{U,O,H}(t) y_U(t)\, dt + B_{U,O,H}(t) \hat{y}_U(t)\, dt + Q^{\dagger}_{U,O,H}(t)\, dw(t), \tag{1.38}$$

for

$$A_{U,O,H}(t) \equiv (A_{U,O} + B_{U,O} A_{U,H}(t)), \tag{1.39}$$
$$B_{U,O,H}(t) \equiv B_{U,O} B_{U,H}(t),$$
$$Q^{\dagger}_{U,O,H}(t) \equiv \left(B_{U,O} Q^{\dagger}_{U,H}(t) + Q^{\dagger}_{U,O} \right).$$

To sum up, under the assumption of an informed's order flow in the form (1.19), it is possible to consider an evolution equation for $\hat{y}_U(t)$ in the form (1.33) provided that (1.26) holds true. In this case, the uninformed nvestor's filtering problem is ruled by the Equations (1.36), (1.34), and (1.38), with coefficients given by (1.37), (1.35), and (1.39), respectively. Hence, introducing the stacked vector $Y_U(t) = (y_U(t), \hat{y}_U(t))^{\intercal}$, the uninformed representative agent can write the following equation for the vector $Y_U(t)$

$$dY_U(t) = A_{U,Y}(t) Y_U(t) dt + Q^{\dagger}_{U,Y}(t) dw(t)$$

where

$$A_{U,Y}(t) = \begin{pmatrix} A_{U,\Phi,H}(t) & B_{U,\Phi,H}(t) \\ A_{U,H}(t) & B_{U,H}(t) \end{pmatrix},$$
$$Q^{\dagger}_{U,Y}(t) = \begin{pmatrix} Q^{\dagger}_{U,\Phi,H}(t) \\ Q^{\dagger}_{U,H}(t) \end{pmatrix},$$

Moreover, the uninformed investor can also rewrite the equation for the observation vector in the form

$$dy_{U,O}(t) = C_{U,Y}(t) Y_U(t) dt + Q^{\dagger}_{U,O,H}(t) dw(t)$$

where

$$C_{U,Y}(t) = \begin{pmatrix} A_{U,O,H}(t) & B_{U,O,H}(t) \end{pmatrix}.$$

Finally, the uninformed representative agent can apply the filtering procedure and we obtain

Proposition 1 *Let us assume that the representative uninformed agent conjectures the representative informed agent's order flow satisfies (1.20). Then, she can write the evolution of $\hat{y}_U(t)$ in the form*

$$d\hat{y}_U(t) = G(t)\hat{y}_U(t)dt + H(t)\, dy_{U,O}(t), \tag{1.40}$$

where the estimate matrices $H(t)$ and $G(t)$ are given by

$$H(t) = \left(\Sigma(t)A^{\intercal}_{U,\Phi,H}(t) + Q^{\dagger}_{U,\Phi,H}(t)Q^{\dagger}_{U,\Phi,H}(t)^{\intercal}\right) M^T \left(Q^{\dagger}_{U,O,H}(t)\left(Q^{\dagger}_{U,O,H}(t)\right)^{\intercal}\right)^{-1} \tag{1.41}$$

and

$$G(t) = (I - H(t)M)(A_{U,\Phi,H}(t) + B_{U,\Phi,H}(t)). \tag{1.42}$$

In addition, the matrix $\Sigma(t) \equiv \mathbf{E}[(y_u(t) - \hat{y}_U(t))(y_u(t) - \hat{y}_U(t))^{\intercal}]$ is the positive solution of the Riccati equation

$$\begin{aligned}\dot{\Sigma}(t) &= A_{U,\Phi,H}(t)\Sigma(t) + \Sigma(t)A^{T}_{U,\Phi,H}(t) + Q^{\dagger}_{U,\Phi,H}(t)\left(Q^{\dagger}_{U,\Phi,H}(t)\right)^{\intercal} \\ &- \left(Q^{\dagger}_{U,\Phi,H}(t)\left(Q^{\dagger}_{U,\Phi,H}(t)\right)^{T} + \Sigma(t)A^{T}_{U,\Phi,H}(t)\right) M^T \left(Q^{\dagger}_{U,O,H}(t)\left(Q^{\dagger}_{U,O,H}(t)\right)^{T}\right)^{-1} \cdot \\ &\cdot M \left(Q^{\dagger}_{U,\Phi,H}(t)\left(Q^{\dagger}_{U,\Phi,H}(t)\right)^{T} + \Sigma(t)A^{T}_{U,\Phi,H}(t)\right)^{T}.\end{aligned} \tag{1.43}$$

As a consequence of Propositions (1), combining (1.33) with (1.29) and (1.42), it is possible to write

$$\begin{aligned}d\hat{y}_U(t) &= (I - H(t)M)(A_{U,\Phi,H}(t) + B_{U,\Phi,H}(t))\hat{y}_U(t)dt + H(t)dy_{U,O}(t) \\ &= (A_{U,\Phi,H}(t) + B_{U,\Phi,H}(t))\hat{y}_U(t)dt \\ &+ H(t)M\left(A_{U,\Phi,H}(t)(y_U(t) - \hat{y}_U(t))dt + Q^{\dagger}_{U,\Phi,H}dw(t)\right)\end{aligned}$$

Now we introduce the symmetric matrix

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{1,2} & b_{2,2} & b_{2,3} \\ b_{1,3} & b_{2,3} & b_{3,3} \end{pmatrix}$$

such that

$$\begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{1,2} & b_{2,2} & b_{2,3} \\ b_{1,3} & b_{2,3} & b_{3,3} \end{pmatrix}^2 = B \circ B.$$

Then, setting

$$M\left(A_{U,\Phi,H}(y_U(t)-\hat{y}_U(t))dt+Q^\dagger_{U,\Phi,H}dw(t)\right)=\begin{pmatrix} d\breve{w}_D \\ d\breve{w}_S \\ d\breve{w}_U \end{pmatrix}\equiv d\breve{w}(t),$$

we have

$$\begin{aligned} dy_{U,O}(t) &= M\left(A_{U,\Phi,H}y_U(t)dt+B_{U,\Phi,H}\hat{y}_U(t)dt+Q^\dagger_{U,\Phi,H}dw(t)\right) \\ &= M(A_{U,\Phi,H}+B_{U,\Phi,H})\hat{y}_U(t)dt+M\left(A_{U,\Phi,H}(y_U(t)-\hat{y}_U(t))dt+Q^\dagger_{U,\Phi,H}dw(t)\right) \\ &= M(A_{U,\Phi,H}+B_{U,\Phi,H})\hat{y}_U(t)dt+BB^{-1}d\breve{w}(t) \\ &= M(A_{U,\Phi,H}+B_{U,\Phi,H})\hat{y}_U(t)dt+Bd\tilde{w}(t), \end{aligned}$$

where the process $\tilde{w}(t)$ given by

$$d\tilde{w}(t)=\begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{1,2} & b_{2,2} & b_{2,3} \\ b_{1,3} & b_{2,3} & b_{3,3} \end{pmatrix}^{-1} d\breve{w}(t) \tag{1.44}$$

generates an information structure $\sigma(\tilde{w}(t))$ which is equivalent to $\mathfrak{F}_U(t)$ and with respect to which $\tilde{w}_U(t)$ is Wiener.

As a consequence, the equation for the uninformed representative agent becomes

$$\begin{aligned} d\hat{y}_U(t) &= (A_{U,\Phi,H}+B_{U,\Phi,H})\hat{y}_U(t)dt+H(t)BB^{-1}d\breve{w}(t) \\ &= \hat{A}_U\hat{y}_U(t)dt+\hat{Q}^\dagger d\tilde{w}(t) \end{aligned} \tag{1.45}$$

where

$$\hat{A}_U=A_{U,\Phi,H}+B_{U,\Phi,H}, \qquad \hat{Q}^\dagger_U=H(t)B$$

Therefore, the composition of Equation (1.10) with (1.18) and (1.45) allows the uninformed representative agent to rewrite her wealth equation (1.9) in terms of only variables that she can observe and the Wiener $\tilde{w}(t)$. This defines a Markov multivariate process and the Separation Principle (see e.g. [13]) can be exploited. Thus the uninformed representative agent's objective function becomes

$$\max_{\Phi_U(\cdot),c_U(\cdot)}\left\{\mathbf{E}_{t,D,\Theta,\Pi,\Upsilon,\Psi_I,\hat{\Pi},\hat{\Theta},\hat{\Upsilon},\hat{\Psi}_I,W_U}\left[\int_t^{+\infty}-e^{-(\rho_U s+\varphi_U c_U(s))}\,ds\right]\right\}, \quad t\geq 0 \tag{1.46}$$

where $\mathbf{E}_{t,D,\Theta,\Pi,\Upsilon,\Psi_I,\hat{\Pi},\hat{\Theta},\hat{\Upsilon},\hat{\Psi}_I,W_U}[\cdot]$ is the conditional expectation operator given the state of random variables D,Θ,Π,Υ,Ψ_I,$\hat{\Pi}$,$\hat{\Theta}$,$\hat{\Upsilon}$,$\hat{\Psi}_I$ and W_U at time t. This makes possible to apply the standard Bellman's procedure.

To this purpose, the uninformed representative agent's putative stock price is rewritten in the form

$$\begin{aligned} P(t) &= p_1 + \hat{p}^\intercal \hat{y}_U(t) + p_{\Psi_U}\Psi_U(t) \\ &= \tilde{p}^\intercal \tilde{y}_U(t) \end{aligned}$$

where

$$\begin{aligned} \hat{p}^\intercal &\equiv (p_D, p_\Pi + p_{\hat{\Pi}}, p_\Theta + p_{\hat{\Theta}}, p_\Upsilon + p_{\hat{\Upsilon}}, 0, p_{\Psi_I} + p_{\hat{\Psi}_I})^\intercal, \\ \tilde{p} &\equiv (p_1, \hat{p}^\intercal, p_{\Psi_U})^\intercal, \quad \tilde{y}_U(t) \equiv (1, \hat{y}_U^\intercal(t), \Psi_U)^\intercal. \end{aligned}$$

It clearly follows

$$dP(t) = \tilde{p}^\intercal d\tilde{y}_U(t), \tag{1.47}$$

and the equation for the dynamics of $\tilde{y}_U(t)$ is given by

$$d\tilde{y}_U(t) = \tilde{A}_U \tilde{y}_U(t)dt + \tilde{Q}_U^\dagger d\tilde{w}(t) + c_{\Phi_U}\Phi_U(t)dt, \tag{1.48}$$

for

$$\tilde{A}_U \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{A}_U & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{Q}_U^\dagger \equiv \begin{pmatrix} 0 \\ \hat{Q}^\dagger \\ 0 \end{pmatrix}, \quad c_{\Phi_U} \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{1.49}$$

Likewise, combining (1.47) with (1.48), the uninformed representative agent write the variation of the stock price in the form

$$dP(t) = \tilde{p}^\intercal \tilde{A}_U \tilde{y}_U(t)dt + \tilde{p}^\intercal \tilde{Q}_U^\dagger d\tilde{w}(t) + \tilde{p}^\intercal c_{\Phi_U}\Phi_U(t)dt.$$

Moreover, since in equilibrium

$$\Psi_U(t) = \frac{1}{\omega}\left(1 - \left(\Upsilon(t) + (1-\omega)\Psi_I(t)\right)\right),$$

the uninformed investor write $\Psi_U(t)$ in the form

$$\Psi_U(t) = e_{\Psi_U}^\intercal \tilde{y}_U(t) = \tilde{y}_U^\intercal(t) e_{\Psi_U}$$

where

$$e_{\Psi_U}^\intercal = (\frac{1}{\omega}, 0, 0, 0, -\frac{1}{\omega}, 0, -\frac{1}{\omega}(1-\omega), 0).$$

As a consequence, the representative uninformed agent write the equation for the instant stock excess return in the form

$$\begin{aligned}dQ(t) &= (k_D^\intercal \tilde{y}_U(t) - r\tilde{p}^\intercal \tilde{y}_U(t))dt + \tilde{p}^\intercal \tilde{A}_U \tilde{y}_U(t)dt + \tilde{p}^\intercal \tilde{Q}_U^\dagger d\tilde{w}(t) + \tilde{p}^\intercal c_{\Phi_U}\Phi_U(t)dt \\ &= (k_D^\intercal - r\tilde{p}^\intercal + \tilde{p}^\intercal \tilde{A}_U)\tilde{y}_U(t)dt + \tilde{p}^\intercal \tilde{Q}_U^\dagger d\tilde{w}(t) + \tilde{p}^\intercal c_{\Phi_U}\Phi_U(t)dt \\ &= \tilde{q}^\intercal \tilde{y}_U(t)dt + \tilde{p}^\intercal \tilde{Q}_U^\dagger d\tilde{w}(t) + \tilde{p}^\intercal c_{\Phi_U}\Phi_U(t)dt\end{aligned}$$

where

$$k_D \equiv (0,1,0,0,0,0,0,0)^\intercal, \quad \tilde{q} \equiv \left(k_D - r\tilde{p}^\intercal + \tilde{p}^\intercal \tilde{A}_U\right)^\intercal,$$

and finally the equation of her wealth becomes

$$\begin{aligned}dW_U(t) &= (rW_U(t) - c_U(t) - \frac{1}{2}\kappa\Phi_U^2(t))\,dt \\ &\quad + \tilde{y}_U^\intercal(t)e_{\Psi_U}(\tilde{q}^\intercal \tilde{y}_U(t)dt + \tilde{p}^\intercal \tilde{Q}^\dagger d\tilde{w}(t) + \tilde{p}^\intercal c_{\Phi_U}\Phi_U(t)dt) \\ &= \left(rW_U(t) - c_U(t) + \tilde{y}_U^\intercal(t)e_{\Psi_U}\left(k_D - r\tilde{p}^\intercal + \tilde{p}^\intercal \tilde{A}\right)\tilde{y}_U(t)\right)dt \\ &\quad + \left(\tilde{y}_U^\intercal(t)e_{\Psi_U}\tilde{p}^\intercal c_{\Phi_U}\Phi_U(t) - \frac{1}{2}\kappa\Phi_U^2(t)\right)dt \\ &\quad + \tilde{y}_U^\intercal(t)e_{\Psi_U}\tilde{p}^\intercal \tilde{Q}^\dagger d\tilde{w}(t)\end{aligned} \tag{1.50}$$

The state dynamical system is then

$$\begin{aligned}d\tilde{y}_U(t) &= \tilde{A}_U\tilde{y}_U(t)dt + \tilde{Q}_U^\dagger d\tilde{w}(t) + c_{\Phi_U}\Phi_U(t)dt \\ dW_U(t) &= \left(rW_U(t) - c_U(t) + \tilde{y}_U^\intercal(t)e_{\Psi_U}\left(k_D - r\tilde{p}^\intercal + \tilde{p}^\intercal \tilde{A}_U\right)\tilde{y}_U(t)\right)dt \\ &\quad + \left(\tilde{y}_U^\intercal(t)e_{\Psi_U}\tilde{p}^\intercal c_{\Phi_U}\Phi_U(t) - \frac{1}{2}\kappa\Phi_U^2(t)\right)dt \\ &\quad + \tilde{y}_U^\intercal(t)e_{\Psi_U}\tilde{p}^\intercal \tilde{Q}_U^\dagger d\tilde{w}(t).\end{aligned}$$

Hence, applying the standard Bellman's procedure, the uninformed representative agent can solve her optimization problem as follows

Proposition 2 *The objective function (1.46) is given by*

$$V(t_0, \tilde{y}_U, W_U) = -e^{-(\rho_U t_0 + \frac{1}{2}\tilde{y}_U^\intercal L_U Y \tilde{y}_U + r\varphi_U W_U + \lambda_U)}, \tag{1.51}$$

where $L_U \equiv (\ell_{j,k})_{j,k=1}^8$ *is a symmetric solution of the algebraic Riccati equation*

$$L_U U_U L_U - L_U V_U - V_U^\intercal L_U - T_U = 0, \tag{1.52}$$

with coefficients

$$U_U \equiv (\tilde{Q}_U^{\dagger} - \frac{1}{r\varphi_U \kappa} c_{\Phi_U} c_{\Phi_U}^{\intercal}), \quad V_U \equiv (A + r\varphi_U Q p e_{\Psi_U}^{\intercal} + r\varphi c_{\Phi_U} c_{\Phi_U}^{\intercal} p e_{\Psi_U}^{\intercal} + \frac{1}{2} r I_7), \tag{1.53}$$

$$T_U \equiv \left(2r\varphi_u e_{\Psi_U}(k_D - r\tilde{p}^{\intercal} + \tilde{p}^{\intercal}\tilde{A}_U)\right),$$

and λ_U *is a real number satisfying*

$$r(1 + \lambda_U - \log(r)) - \rho_U - \frac{1}{2}\mathbf{tr}\left(\left(\tilde{Q}_U^{\dagger}\right)^{\intercal} L_U \left(\tilde{Q}_U^{\dagger}\right)\right) = 0. \tag{1.54}$$

In addition, the representative uninformed agent's optimal order flow for the stock and consumption are given by

$$\mathring{\Phi}_U(t) = -\frac{(L_U e_{\Phi_U} + r\varphi_U e_{\Psi_U} p^{\intercal} e_{\Phi})^{\intercal}}{r\varphi_U \kappa} Y(t), \quad t \geq t_{0,} \tag{1.55}$$

$$\mathring{c}_U(t) = \frac{\frac{1}{2}\tilde{y}_U^{\intercal}(t) L_U \tilde{y}_U(t) + r\varphi_U W_U(t) + \lambda_U - \ln(r)}{\varphi_U}, \quad t \geq t_{0,} \tag{1.56}$$

respectively, where $(\tilde{y}_U(t)(t), W_U(t))$ *is the solution of (1.48), (1.50), corresponding to the choice of the optimal control* $(\mathring{\Phi}_U(t), \mathring{c}_U(t))$ *and the state of the random variables* $\tilde{y}_U(t)$, $W_U(t)$ *at time* t_0.

1.3 Informed Representative Agent's Optimization Problem

The representative informed agent holds complete information. On this basis, she aims to write the equation for $dQ(t)$

$$dQ(t) = (D(t) - rP(t))\, dt + dP(t)$$

in terms of the variables and the noises of the economy she can observe. On the other hand, $dQ(t)$ depends on $dP(t)$ and the informed conjectures that $P(t)$ is set by the uninformed representative agent according to the linear equation

$$P(t) = \tilde{p}_I^{\intercal} \tilde{y}_U(t)$$

where $\tilde{y}_U(t)$ is the uninformed investor's extended estimate vector and $\tilde{p}_I^{\intercal}$ is the vector of the coefficients of $P(t)$ w.r.t. $\tilde{y}_U(t)$, conjectured by the informed. In addition, the informed representative agent conjectures that the representative uninformed agent updates her estimates according to the equation

$$d\tilde{y}_U(t) = G_I \tilde{y}_U(t)dt + H_I dy_{U,O}(t).$$

Now, introducing the insider's state vector $y_I(t) \equiv (D(t), \Pi(t), \Theta(t), \Upsilon(t), \Psi_U(t), \Psi_I(t))^\intercal$, which, on account of the differential form of the market clearing condition

$$\begin{aligned} d\Psi_U(t) &= -\omega^{-1}((1-\omega)d\Psi_I(t) + d\Upsilon(t)) \\ &= -\omega^{-1}((1-\omega)\Phi_I(t) + \Theta(t))dt \end{aligned}$$

fulfills

$$dy_I(t) = A_I y_I(t)dt - e_{\Phi_I}\Phi_I(t)dt + Q_I^\dagger dw(t),$$

for

$$A_I \equiv \begin{pmatrix} -\alpha_D & 1 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_\Pi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_\Theta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\omega^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$e_{\Phi_I} \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -(1-\omega)\omega^{-1} \\ 1 \end{pmatrix}, \quad Q_I^\dagger \equiv \begin{pmatrix} \sigma_{D,D} & \sigma_{D,\Pi} & 0 \\ 0 & \sigma_{\Pi,\Pi} & 0 \\ \sigma_{D,\Theta} & 0 & \sigma_{\Theta,\Theta} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

we have

$$\begin{aligned} dy_{U,O}(t) &= M_I dy_I(t) \\ &= M_I \left(A_I y_I(t)dt - e_{\Phi_I}\Phi_I(t)dt + Q_I^\dagger dw(t) \right) \\ &= M_I A_I y_I(t)dt - M_I e_{\Phi_I}\Phi_I(t)dt + M_I Q_I^\dagger dw(t), \end{aligned}$$

where

$$M_I \equiv M_U,$$

and

$$\begin{aligned} d\tilde{y}_U(t) &= G_I \tilde{y}_U(t)dt + H_I dy_{U,O}(t) \\ &= H_I M_I A_I y_I(t)dt + G_I \tilde{y}_U(t)dt - H_I M_I e_{\Phi_I}\Phi_I(t)dt + H_I M_I Q_I^\dagger dw(t). \end{aligned}$$

Therefore, introducing the insider's stacked vector $Y^\intercal(t) \equiv (\tilde{y}_U^\intercal(t), y_I^\intercal(t))^\intercal$ we have

$$dY(t) = A_Y Y(t)dt - e_{Y,\Phi_I}\Phi_I(t)dt + Q_Y^\dagger dw(t), \tag{1.57}$$

where

$$A_Y \equiv \begin{pmatrix} G_I & H_I M_I A_I \\ 0 & A_I \end{pmatrix}, \quad e_{Y,\Phi_I} \equiv \begin{pmatrix} e_{\Phi_I} \\ H_I M_I e_{\Phi_I} \end{pmatrix}, \quad Q_Y^\dagger \equiv \begin{pmatrix} Q_I^\dagger \\ H_I M_I Q_I^\dagger \end{pmatrix}.$$

Moreover, we can write

$$P(t) = \tilde{p}_I^\intercal \tilde{y}_U(t) = p_Y^\intercal Y(t),$$

and

$$\begin{aligned} \Psi_I(t) &= (1-\omega)^{-1}(1 - (\Upsilon(t) + \omega\Psi_U(t))) \\ &= e_{\Psi_I}^\intercal Y(t) = Y^\intercal(t) e_{\Psi_I} \end{aligned}$$

for

$$p_Y \equiv \begin{pmatrix} 0 \\ p_I \end{pmatrix},$$

and

$$e_{\Psi_I}^\intercal = ((1-\omega)^{-1}, 0, 0, 0, -(1-\omega)^{-1}, 0, 0, 0, 0, 0, 0, 0, -(1-\omega)^{-1}\omega, 0)$$

As a consequence, we have

$$\begin{aligned} dQ(t) &= (e_{Y,D}Y(t) - r p_Y^\intercal Y(t))\,dt + p_Y^\intercal dY(t) \\ &= (e_{Y,D} - r p_Y^\intercal) Y(t)\,dt + p_Y^\intercal (A_Y Y(t)dt - e_{Y,\Phi_I}\Phi_I(t)dt + Q_Y^\dagger dw(t)) \\ &= (e_{Y,D} - r p_Y^\intercal + p_Y^\intercal A_Y) Y(t)\,dt - p_Y^\intercal e_{Y,\Phi_I}\Phi_I(t)dt + p_Y^\intercal Q_Y^\dagger dw(t) \end{aligned}$$

where

$$e_{Y,D} = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

and

$$\begin{aligned} dW_I(t) &= (rW_I(t) - c_I(t) - \frac{1}{2}\kappa\Phi_I^2(t))\,dt + \Psi_I(t)dQ(t) \qquad (1.58) \\ &= (rW_I(t) - c_I(t) - \frac{1}{2}\kappa\Phi_I^2(t))\,dt \\ &+ Y^\intercal(t)e_{\Psi_I}((e_{Y,D} - r p_Y^\intercal + p_Y^\intercal A_Y)Y(t)\,dt - p_Y^\intercal e_{Y,\Phi_I}\Phi_I(t)dt + p_Y^\intercal Q_Y^\dagger dw(t)) \\ &= (rW_I(t) - c_I(t) + Y^\intercal(t)e_{\Psi_I}(e_{Y,D} - r p_Y^\intercal + p_Y^\intercal A_Y)Y(t)dt \\ &+ \left(-Y^\intercal(t)e_{\Psi_I} p_Y^\intercal e_{Y,\Phi_I}\Phi_I(t) - \frac{1}{2}\kappa\Phi_I^2(t)\right)\,dt \\ &+ Y^\intercal(t)e_{\Psi_I} p_Y^\intercal Q_Y^\dagger dw(t). \end{aligned}$$

The constraining system is then

$$\begin{aligned} dW_I(t) &= (rW_I(t) - c_I(t) + Y^\intercal(t) e_{\Psi_I}(e_{Y,D} - rp_Y^\intercal + p_Y^\intercal A_Y) Y(t) dt \\ &\quad + \left(-Y^\intercal(t) e_{\Psi_I} p_Y^\intercal e_{Y,\Phi_I} \Phi_I(t) - \frac{1}{2}\kappa \Phi_I^2(t) \right) dt \\ &\quad + Y^\intercal(t) e_{\Psi_I} p_Y^\intercal Q_Y^\dagger dw(t) \\ dY(t) &= A_Y Y(t) dt - e_{Y,\Phi_I}\Phi_I(t) dt + Q_Y^\dagger dw(t). \end{aligned}$$

Hence, the composition of Equation(1.10) with (1.17) and (1.57) allows the uninformed representative agent to rewrite her wealth equation (1.9) in terms of all variables that the informed investor can observe. This defines a Markov multivariate process and the informed representative agent's objective function becomes

$$\max_{\Phi_I(\cdot), c_I(\cdot)} \left\{ \mathbf{E}_{t,D,\Theta,\Pi,\Upsilon,\Psi_U,\hat{\Pi},\hat{\Theta},\hat{\Upsilon},\hat{\Psi}_U,W_I} \left[\int_t^{+\infty} -e^{-(\rho_I s + \varphi_I c_I(s))} ds \right] \right\}, \quad t \geq 0 \tag{1.59}$$

where $\mathbf{E}_{t,D,\Theta,\Pi,\Upsilon,\Psi_U,\hat{\Pi},\hat{\Theta},\hat{\Upsilon},\hat{\Psi}_U,W_I}[\cdot]$ is the conditional expectation operator given the state of random variables D,Θ,Π,Υ,Ψ_U,$\hat{\Pi}$,$\hat{\Theta}$,$\hat{\Upsilon}$,$\hat{\Psi}_U$ and W_I at time t. Therefore, we can directly apply the standard Bellman's procedure. The insider's optimization problem is then solved as follows.

Proposition 3 *The objective function (1.59) is given by*

$$V(t_0, Y, W_I) = -e^{-(\rho_I t_0 + \frac{1}{2} Y^\intercal L_I Y + r\varphi_I W_I + \lambda_I)}, \tag{1.60}$$

where $L_I \equiv (\ell_{j,k})_{j,k=0}^{13}$ is a symmetric solution of the algebraic Riccati equation

$$L_I U_I L_I - L_I V_I - V_I^\intercal L_I - T_I = 0, \tag{1.61}$$

with coefficients

$$\begin{aligned} U_I &\equiv (Q - \frac{1}{r\varphi\kappa} e_{Y,\Phi_I} e_{Y,\Phi_I}^\intercal), \\ V_I &\equiv (A + r\varphi Q p e_{\Psi_I}^\intercal + r\varphi e_{Y,\Phi_I} e_{Y,\Phi_I}^\intercal p e_{\Psi_I}^\intercal + \frac{1}{2} r I_{13}), \\ T_I &\equiv (2r\varphi e_{\Psi_I}(e_D - rp^\intercal + p^\intercal A)), \end{aligned}$$

for a real number λ_I satisfying

$$r(1 + \lambda_I - \log(r)) - \rho_I - \frac{1}{2}\mathbf{tr}\left(\left(Q_Y^\dagger\right)^\intercal L_I \left(Q_Y^\dagger\right) \right) = 0. \tag{1.62}$$

In addition, the uninformed investors' optimal demand for the stock and consumption are given by

$$\mathring{\Phi}_I(t) = -\frac{(Le_{Y,\Phi_I} + r\varphi e_{\Psi_I} p^\intercal e_{Y,\Phi_I})^\intercal}{r\varphi\kappa} Y(t), \quad t \geq t_{0}, \tag{1.63}$$

$$\mathring{c}_I(t) = \frac{\frac{1}{2} Y^\intercal(t) L_I Y(t) + r\varphi_I W_I(t) + \lambda_I - \ln(r)}{\varphi_I}, \quad t \geq t_{0}, \tag{1.64}$$

respectively, where $(Y(t), W_I(t))$ *is the solution of (1.57), (1.58), corresponding to the choice of the optimal control* $(\mathring{\Phi}_I(t), \mathring{c}_I(t))$ *and the state of the random variables* Y, W_I *at time* t_0.

Conclusions

In this paper we have deepened the study of Wang's model by introducing a parameter of overreaction of non rational investors to public dividend surprise. Our results constitute a fresh contribution to the theory of economic equilibrium in incomplete financial markets under asymmetric information. First,we have discovered the existence of Pareto efficient equilibria additional to the one revealed by Wang in his paper. Second, we have discovered new equilibrium candidates with a strategic flavor. With respect to the informationally semi-strang form efficient price determined by Wang, the prices characterizing such strategic equilibrium candidates yield a higher discount for holding the stock, and exhibit both a higher sensitivity to the stock supply shocks and some extent of informational inefficiency. Moreover, while under low stock supply volatility and investors' risk aversion we have found no strategic equilibrium candidate Pareto dominating Wang's one, under high market risk perception equilibrium candidates occur in which the investors of the two groups trading both strategically achieve both a higher utility. The economical interpretation seems to us intriguing: as rational investors' perception of market risk is high, they get a higher utility from a strategic trading policy which leads to informationally inefficient equilibria. Finally, by introducing transaction costs in investor's wealth equations, we discover the existence of candidate equilibrium stock prices, whose research changes significantly from both the theoretical point of view and the computational one.

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