Soluzioni del tutorato di Statistica 1 del 04/03/2009
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Esercizio 1.
Siano $X_1, ..., X_n$ v.a. i.i.d. con densità:

$$f_X(x) = x^{-2} 1_{(1,\infty)}(x)$$

$$E[X_1] = \int_1^{\infty} x x^{-2} dx = \int_1^{\infty} \frac{1}{x} dx \rightarrow +\infty$$

Dunque $E[X_1]$ non esiste.

$$E[Y] = \int_{-\infty}^{\infty} y f_Y(y) dy$$

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

Per $y > 1$

$F_Y(y) = P(Y \leq y) = 1 - P(min\{X_1, .., X_n\} > y) = 1 - P(X_1 > y)..P(X_n > y) =$

$$= 1 - [P(X > y)]^n = 1 - [\int_y^{+\infty} x^{-2} dx]^n = 1 - \frac{1}{y^n}.$$

Allora:

$$f_Y(y) = \frac{n}{y^{n+1}}$$

e:

$$E[Y] = \int_1^{+\infty} y \frac{n}{y^{n+1}} dy = \frac{n}{n-1}.$$

Esercizio 2.
$X_i \sim Po(\lambda)$, $S_n = \sum_{i=1}^n X_i$;

1.

$$E[S_n] = E[\sum_{i=1}^n X_i] = \sum_{i=1}^n E[X_i] = n\lambda$$

$$Var[S_n] = Var[\sum_{i=1}^n X_i] = \sum_{i=1}^n Var[X_i] = n\lambda$$

2. Usando la disuguaglianza di Markov:

$$P(S_{100} > 440) < \frac{400}{440} = \frac{10}{11}.$$

Esercizio 3.

X e Y sono due variabili casuali discrete con funzione di densità:

$$f(x,y) = \frac{c}{(x+y-1)(x+y)(x+y+1)}$$

per x,y=1,2,...

$$P(X=x) = \sum_{y=1}^{+\infty} P(X=x, Y=y) = \sum_{y=1}^{+\infty} \frac{c}{(x+y-1)(x+y)(x+y+1)} =$$

$$\sum_{y=1}^{+\infty} \frac{c}{2} \left\{ \frac{1}{(x+y-1)(x+y)} - \frac{1}{(x+y+1)(x+y)} \right\} = \frac{c}{2} \left\{ \frac{1}{x} - \frac{1}{x+1} \right\}$$

Per calcolare c:

$$\sum_{x=1}^{+\infty} \frac{c}{2} \left\{ \frac{1}{x} - \frac{1}{x+1} \right\} = 1$$

inoltre so che:

$$\sum_{x=1}^{+\infty} \left\{ \frac{1}{x} - \frac{1}{x+1} \right\} = 1$$

Allora

$$c = 2$$

Esercizio 4.

Sia $\Omega = \{\omega_1, \omega_2, \omega_3\}$, lo spazio degli eventi, siano $P(\omega_1) = P(\omega_2) = P(\omega_3) = \frac{1}{3}$. Siano $X, Y, Z : \Omega \longrightarrow R$:

$$\begin{array}{ccc} X(\omega_1) = 1 & X(\omega_2) = 2 & X(\omega_3) = 3 \\ Y(\omega_1) = 2 & Y(\omega_2) = 3 & Y(\omega_3) = 1 \\ Z(\omega_1) = 2 & Z(\omega_2) = 2 & Z(\omega_3) = 1 \end{array} \quad (1)$$

$P(X=k)=\frac{1}{3}$ per $k=1,2,3$, $P(Y=k)=\frac{1}{3}$ per $k=1,2,3$

$$\begin{array}{ccc} W_1(\omega_1)=3 & W_1(\omega_2)=5 & W_1(\omega_3)=4 \\ W_2(\omega_1)=2 & W_2(\omega_2)=6 & W_2(\omega_3)=3 \\ W_3(\omega_1)=\dfrac{1}{2} & W_3(\omega_2)=\dfrac{2}{3} & W_3(\omega_3)=3 \end{array} \tag{2}$$

$P(W_1=X+Y)=\frac{1}{3}$, $P(W_2=XY)=\frac{1}{3}$, $P(W_3=X/Y)=\frac{1}{3}$

$$\begin{array}{ccc} P(Y=1|Z=1)=1 & P(Y=2|Z=1)=0 & P(Y=3|Z=1)=0 \\ P(Y=1|Z=2)=0 & P(Y=2|Z=2)=\dfrac{1}{2} & P(Y=3|Z=2)=\dfrac{1}{2} \end{array} \tag{3}$$

$$\begin{array}{ccc} P(Z=1|Y=1)=1 & P(Z=1|Y=2)=0 & P(Z=1|Y=3)=0 \\ P(Z=2|Y=1)=0 & P(Z=2|Y=2)=1 & P(Z=2|Y=3)=1 \end{array} \tag{4}$$