Tutorato 6 - ICA
Soluzioni

1. $$\lim_{n\to\infty} \frac{\sqrt{n^3+9n^2}-\sqrt{n^4+1}}{n^2+2} = \lim_{n\to\infty} \frac{n^2\left(\sqrt{\frac{1}{n}+\frac{9}{n^2}}-\sqrt{1+\frac{1}{n^4}}\right)}{n^2\left(1+\frac{2}{n^2}\right)} = -1$$

2. $$\lim_{n\to\infty} \sqrt[n]{n^4+1} = 1$$

3. $$\lim_{n\to\infty} \frac{n}{2^n-3^n} = \lim_{n\to\infty} \frac{n}{2^n\left(1-\left(\frac{3}{2}\right)^n\right)} = 0$$

4. $$\lim_{n\to\infty} \frac{n^2}{n!} = 0$$

5. $$\lim_{n\to\infty} \frac{n^{20}+4n^4+1}{n!} = \lim_{n\to\infty} \frac{n^{20}}{n!}+\frac{4n^4}{n!}+\frac{1}{n!} = 0$$

6. $$\lim_{n\to\infty} \left(1+\frac{1}{n!}\right)^n = \lim_{n\to\infty} \left(1+\frac{1}{n!}\right)^{n!\,\frac{n}{n!}} = 1$$

7. $$\lim_{n\to\infty} n\left(\sqrt{1+\frac{2}{n^2}}-\sqrt{1-\frac{4}{n}}\right) =$$
$$= \lim_{n\to\infty} n\left(\sqrt{1+\frac{2}{n^2}}-\sqrt{1-\frac{4}{n}}\right)\frac{\sqrt{1+\frac{2}{n^2}}+\sqrt{1-\frac{4}{n}}}{\sqrt{1+\frac{2}{n^2}}+\sqrt{1-\frac{4}{n}}} =$$
$$= \lim_{n\to\infty} \frac{n\left(1+\frac{2}{n^2}-1-\frac{4}{n}\right)}{\sqrt{1+\frac{2}{n^2}}+\sqrt{1-\frac{4}{n}}} = \lim_{n\to\infty} \frac{\frac{2}{n}-4}{\sqrt{1+\frac{2}{n^2}}+\sqrt{1-\frac{4}{n}}} = -2$$

8. $$\lim_{n\to\infty} \frac{\sin n}{n} = 0 \qquad \text{perché} \qquad \frac{-1}{n} \leqslant \frac{\sin n}{n} \leqslant \frac{1}{n}$$

9. $$\lim_{n\to\infty} \frac{(n+1)^6-(n-1)^6}{(n+1)^5+(n-1)^5} =$$
$$= \lim_{n\to\infty} \frac{n^6+6n^5+c_4n^4+c_3n^3+c_2n^2+6n+1-(n^6-6n^5+c_4n^4-c_3n^3+c_2n^2-6n+1)}{n^5+5n^4+d_3n^3+d_2n^2+5n+1+n^5-5n^4+d_3n^3-d_2n^2+5n-1} =$$
$$= \lim_{n\to\infty} \frac{12n^5+2c_3n^3+12n}{2n^5+2d_3n^3+10n} = 6$$

10. $$\lim_{n\to\infty}\left(1+\frac{1}{n!}\right)^{n^n} = \lim_{n\to\infty}\left(1+\frac{1}{n!}\right)^{n!\cdot\frac{n^n}{n!}} = \infty$$

11. $$\lim_{n\to\infty}\frac{\log(n^3)}{\log(n^3+3n^2)} = \lim_{n\to\infty}\frac{\log(n^3)}{\log\left(n^3\left(1+\frac{3}{n}\right)\right)} =$$

$$= \lim_{n\to\infty}\left(\frac{\log n^3+\log\left(1+\frac{3}{n}\right)}{\log n^3}\right)^{-1} = \lim_{n\to\infty}\left(1+\frac{\log\left(1+\frac{3}{n}\right)}{\log n^3}\right)^{-1} = 1$$

12. $$\lim_{n\to\infty}\frac{n^2(\log n)^2}{\sqrt{n^5}+1} = 0$$

perché $$0 \leqslant \frac{n^2(\log n)^2}{\sqrt{n^5}+1} \leqslant \frac{n^2(\log n)^2}{\sqrt{n^5}} = \frac{(\log n)^2}{n^{1/2}} \to 0 \qquad \text{per } n\to\infty$$