

Tutorato 5 - ICA
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1. Dimostrare per induzione che

$$(a) \quad 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + (n-1) \cdot n = \frac{(n-1)n(n+1)}{3}$$

$$(b) \quad \prod_{k=2}^n \left(1 - \frac{1}{k^2}\right) = \frac{n+1}{2n} \quad \text{con } n \geq 2$$

2. Determinare l'estremo superiore e l'estremo inferiore dei seguenti insiemi di numeri reali e dire se si tratta del massimo o del minimo

$$(a) \quad \left\{ \frac{n}{n+1} \mid n \in \mathbb{N} \right\}$$

$$(b) \quad \left\{ \frac{x}{x^2+1} \mid x \in \mathbb{R} \right\}$$

$$(c) \quad \left\{ \frac{x}{x+1} \mid x \in (-2, -1) \right\}$$

$$(d) \quad \left\{ \frac{3n^2}{4n+1} \mid n \in \mathbb{N} \right\}$$

$$(e) \quad \left\{ (-1)^n \frac{1}{n+1} \mid n \in \mathbb{N} \right\}$$

$$(f) \quad \{x \in \mathbb{R} \mid x^3 < 8\}$$

$$(g) \quad \{x \in \mathbb{R} \mid x+1 \geq 3x-2\}$$

$$(h) \quad \{x \in \mathbb{R} \mid x+5 \leq 3x+4\}$$

$$(i) \quad \{x \in \mathbb{Q} \mid x^2 \leq 2\}$$

$$(j) \quad (0, 1] \cup [2, 3]$$

$$(k) \quad (-\infty, 4] \cap (1, 5)$$

$$(l) \quad \left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots \right\}$$

$$(m) \quad \left\{ -1, \frac{1}{2}, -\frac{1}{3}, \dots, \frac{(-1)^n}{n}, \dots \right\}$$

$$(n) \quad \{x \in \mathbb{R} \mid x^2 \in \mathbb{Q}\}$$