Tutorato 8 - ICA
Soluzioni

a) Calcolare i seguenti limiti

1. $\lim_{x\to 0} \frac{\log(2-\cos x)}{\sin^2 x} = \lim_{x\to 0} \frac{\log(1+1-\cos x)}{1-\cos x} \cdot \frac{1-\cos x}{x^2} \cdot \frac{x^2}{\sin^2 x} = \frac{1}{2}$

2. $\lim_{x\to\infty}(x - \sin^2 x \log x) = \infty$

perché $x - \sin^2 x \log x \geqslant x - \log x = \log x \left(\frac{x}{\log x} - 1\right) \to \infty$ per $x \to \infty$

3. $\lim_{x\to\infty} \frac{\sqrt{5+\cos x}}{x^2+1} = 0$

perché $0 \leqslant \frac{\sqrt{5+\cos x}}{x^2+1} \leqslant \frac{\sqrt{6}}{x^2+1} \to 0$ per $x \to \infty$

4. $\lim_{x\to 0}\left[\frac{1}{x\tan x} - \frac{1}{x\sin x}\right] = \lim_{x\to 0}\left[\frac{\cos x}{x\sin x} - \frac{1}{x\sin x}\right] = \lim_{x\to 0} \frac{\cos x - 1}{x\sin x} \cdot \frac{x}{x} = -\frac{1}{2}$

5. $\lim_{x\to 0} \frac{\log(\tan^4 x+1)}{e^{2\sin^4 x}-1} = \lim_{x\to 0} \frac{\log(\tan^4 x+1)}{2\tan^4 x} \cdot \frac{2\tan^4 x}{e^{2\sin^4 x}-1} =$

$= \lim_{x\to 0} \frac{\log(\tan^4 x+1)}{2\tan^4 x} \cdot \frac{2\sin^4 x}{\cos^4 x\left(e^{2\sin^4 x}-1\right)} = \frac{1}{2}$

6. $\lim_{x\to 0} \frac{e^{\sqrt{\sin x}}-1}{\sqrt{x}} = \lim_{x\to 0} \frac{e^{\sqrt{\sin x}}-1}{\sqrt{\sin x}} \cdot \frac{\sqrt{\sin x}}{\sqrt{x}} = \lim_{x\to 0} \frac{e^{\sqrt{\sin x}}-1}{\sqrt{\sin x}} \cdot \sqrt{\frac{\sin x}{x}} = 1$

b) Calcolare la derivata delle seguenti funzioni:

1. $\dfrac{x^2-1}{x(x+2)} \longrightarrow \dfrac{2(x^2+x+1)}{x^2(x+2)^2}$

2. $\dfrac{3x^5-2x^3+5}{x^4-3x^2+3x} \longrightarrow \dfrac{-15+30x-32x^3+6x^4+36x^5-25x^6+3x^8}{(x^4-3x^2+3x)^2}$

3. $x^2-3x+2 \longrightarrow 2x-3$

4. $\sqrt[3]{1-3x}-x \longrightarrow -x(1-3x)^{-\frac{2}{3}}-1$

5. $x^2\sin x \longrightarrow 2x\sin x + x^2\cos x$

6. $e^x\cos x \longrightarrow e^x\cos x - e^x\sin x$