Università degli Studi Roma Tre
Dipartimento di Matematica

Recupero II Esonero
FS1, Dinamica e Termodinamica
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SOLUZIONI

$u_{x1} = 100 \text{ cm/s} \qquad u_{y1} = 0 \text{ cm/s} \qquad u_{x2} = 100 \cos\alpha = 70.7 \text{ cm/s} \qquad u_{y2} = 100 \sin\alpha = 70.7 \text{ cm/s}$

$$\begin{cases} m_1 u_{x1} + m_2 u_{x2} = m_1 v_{x1} + m_2 v_{x2} \\ v_{x1} - v_{x2} = -0.75(100 - 70.7)\text{ cm/s} \end{cases}$$

$v_{x1} = 65.8 \text{ cm/s} \qquad v_{x2} = 87.8 \text{ cm/s}$

1) $v_{y1} = u_{y1} = 0 \text{ cm/s} \qquad v_{y2} = u_{y2} = 70.7 \text{ cm/s}$

$\mathbf{v}_1 = 65.8\,\mathbf{i} \text{ cm/s}$

$\mathbf{v}_2 = (87.8\,\mathbf{i} + 70.7\,\mathbf{j})\text{ cm/s} \qquad v_2 = 112.7 \text{ cm/s}$

$$\varphi = \tan^{-1}\frac{70.7}{87.8} = 38° 51'$$

2-a)

$$\frac{T_{max}}{T_{min}} = \frac{T_2}{T_1} = 4$$

$$U_{max} - U_{min} = U_2 - U_1 = 3\, p_1\, V_1 \frac{c_V}{R} = 1.07 \cdot 10^4 \text{ J}$$

2-b)

$$V_{max} = V_3 = V_1\, 2^{(1 + 2\,c_V/R)} = 0.787 \text{ m}^3$$

$$p_{min} = p_3 = p_1\, 2^{-(1 + 2\,c_V/R)} = 1.64 \cdot 10^3 \text{ Pa}$$

2-c)

$$L = Q_{1\to2} + Q_{3\to1} = p_1\, V_1 \left(1 + \frac{2\,c_V}{R}\right)\left(\frac{3}{2} - \ln 2\right) = 6.81 \cdot 10^3 \text{ J}$$

$$\eta = 1 + \frac{Q_{3\to1}}{Q_{1\to2}} = 1 - \frac{2}{3}\ln 2 = 53.8\ \%$$

3-a)

$p_0 = n\,R\,T_0 / V_0 \quad \Rightarrow \quad p_0\, A = n\,R\,T_0\, A / V_0$

$p_0\, A = mg \qquad V_0 / A = z = h$

$$h = \frac{n\,R\,T_0}{mg} = 0.50 \text{ m}$$

$p\,A = n\,R\,T\,A/V = n\,R\,T/z$

$$m\frac{d^2z}{dt^2} = \frac{n\,R\,T}{z} - mg$$

$$TV^{\gamma-1} = T_0 V_0^{\gamma-1} \quad \Rightarrow \quad T = T_0\left(\frac{z_0}{z}\right)^{\gamma-1}$$

Calcolo dei lavori

$$mg\,(z_0 - z)$$

$$\int_{V_0}^{V} p\,dV = n\,R\int_{V_0}^{V}\frac{T}{V}\,dV = n\,R\,T_0\,z_0^{\gamma-1}\int_{z_0}^{z} z^{-\gamma}dz = \frac{n\,R\,T_0}{1-\gamma}\left[\left(\frac{z}{z_0}\right)^{1-\gamma} - 1\right]$$

$$mg\,(z_0 - z) + \frac{n\,R\,T_0}{1-\gamma}\left[\left(\frac{z}{z_0}\right)^{1-\gamma} - 1\right] = \frac{1}{2}\,m\left(\frac{dz}{dt}\right)^2$$

$$v = \frac{dz}{dt} = \sqrt{2g\,(z_0 - z) - \frac{2\,n\,R\,T_0}{m(\gamma-1)}\left[\left(\frac{z_0}{z}\right)^{\gamma-1} - 1\right]}$$

3-b) per $z = z_1 \quad \Rightarrow \quad \dfrac{d^2z}{dt^2} = 0$

$$z_1 = z_0\left(\frac{n\,R\,T_0}{m\,g\,z_0}\right)^{1/\gamma} = 0.53\ m$$

$$v_1 = \sqrt{2g\,z_0\left[1 - \left(\frac{n\,R\,T_0}{m\,g\,z_0}\right)^{1/\gamma}\right] - \frac{2\,n\,R\,T_0}{m(\gamma-1)}\left[\left(\frac{m\,g\,z_0}{n\,R\,T_0}\right)^{1-1/\gamma} - 1\right]} = 0.36\ m/s$$
