Università degli Studi Roma Tre
Dipartimento di Matematica

II Prova di Esonero
FS1, Dinamica e Termodinamica
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SOLUZIONI

1-a)

$$v_0 = \sqrt{2gh}$$
$$v_1 = \sqrt{2ghq}$$
$$v_2 = \sqrt{2ghq^2}$$
.
.
$$v_{n-1} = \sqrt{2ghq^{n-1}}$$
$$v_n = \sqrt{2ghq^n}$$

$$c = \frac{v_1}{v_0} = \frac{v_2}{v_1} = ... = \frac{v_n}{v_{n-1}} = \sqrt{q} = 0.9$$

1-b)

$$\Delta E_1 = \frac{1}{2}m v_1^2 - \frac{1}{2}m v_0^2 = mgh(1-q) = 0.04\ \text{J}$$
$$\Delta E_2 = \frac{1}{2}m v_2^2 - \frac{1}{2}m v_1^2 = mgh(1-q)q = 0.03\ \text{J}$$
.
.
$$\Delta E_n = \frac{1}{2}m v_n^2 - \frac{1}{2}m v_{n-1}^2 = mgh(1-q)q^{n-1}$$

1-c)

$$\frac{d^2 z}{dt^2} = -g, \qquad \frac{dz}{dt} = -gt + v_1, \qquad z = -\frac{1}{2}gt^2 + v_1 t$$

$$z(t_1) = 0 \quad \Rightarrow \quad t_1 = 2\sqrt{\frac{2hq}{g}} = 0.96\ \text{s}$$
$$t_2 = 2\sqrt{\frac{2h}{g}}\ q = 0.87\ \text{s}$$
.
.
$$t_n = 2\sqrt{\frac{2h}{g}}\ q^{n/2}$$

2-a)

$$L = Q_1 + Q_2 + Q_3 \geq 0 \qquad \Rightarrow \qquad Q_3 \geq -Q_1 - Q_2$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} \leq 0 \qquad \Rightarrow \qquad Q_3 \leq -T_3\left(\frac{Q_1}{T_1} + \frac{Q_2}{T_2}\right)$$

$$-48\,\text{cal} \leq Q_3 \leq -30\,\text{cal} \qquad \Rightarrow \qquad -201\,\text{J} \leq Q_3 \leq -126\,\text{J}$$

2-b)

$$0 \leq L \leq \left(1 - \frac{T_3}{T_1}\right)Q_1 + \left(1 - \frac{T_3}{T_2}\right)Q_2$$

$$0 \leq L \leq 75.2\,\text{J}$$

2-c)

$$\eta = L/Q_1$$

$$0 \leq \eta \leq \left(1 - \frac{T_3}{T_1}\right) + \left(1 - \frac{T_3}{T_2}\right)\frac{Q_2}{Q_1}$$

$$0 \leq \eta \leq 32.7\,\%$$

3-a)

$$\Delta U_1 + \Delta U_2 = 0 \qquad \Rightarrow \qquad \Delta U_1 = Q_1 \qquad \Rightarrow \qquad \Delta U_2 = Q_2 \qquad \Rightarrow \qquad Q_1 = -Q_2$$

per c costante

$$m_1 \int_{T_1}^{T_f} c\, dT + m_2 \int_{T_2}^{T_f} c\, dT = 0$$

$$m_1\, c\left(T_f - T_1\right) + m_2\, c\left(T_f - T_2\right) = 0$$

$$T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = 289.2\ \text{K}$$

per $c = c_0 + \alpha T$

$$m_1 \int_{T_1}^{T_f} \left(c_0 + \alpha T\right) dT + m_2 \int_{T_2}^{T_f} \left(c_0 + \alpha T\right) dT = 0$$

$$m_1 c_0 \left(T_f - T_1\right) + \frac{1}{2} m_1 \alpha \left(T_f^2 - T_1^2\right) + m_2 c_0 \left(T_f - T_2\right) + \frac{1}{2} m_2 \alpha \left(T_f^2 - T_2^2\right) = 0$$

$$T_f = \sqrt{\left(\frac{c_0}{\alpha}\right)^2 + \frac{2 c_0 \left(m_1 T_1 + m_2 T_2\right) + \alpha \left(m_1 T_1^2 + m_2 T_2^2\right)}{\alpha \left(m_1 + m_2\right)}} - \frac{c_0}{\alpha} = 289.4\ \text{K}$$

3-b)

$$\Delta U_1 + \Delta U_2 = 0 \quad \Rightarrow \quad \Delta U_1 = Q_1 \quad \Rightarrow \quad \Delta U_2 = Q_2 \quad \Rightarrow \quad Q_1 = -Q_2$$

per c costante

$$\Delta S = m_1 \int_{T_1}^{T_f} \frac{c}{T}\, dT + m_2 \int_{T_2}^{T_f} \frac{c}{T}\, dT$$

$$\Delta S = m_1\, c \ln\left(T_f / T_1 \right) + m_2\, c \ln\left(T_f / T_2 \right) = 82.81\ J/K$$

per $c = c_0 + \alpha T$

$$\Delta S = m_1 \int_{T_1}^{T_f} \left(\frac{c_0 + \alpha T}{T} \right) dT + m_2 \int_{T_2}^{T_f} \left(\frac{c_0 + \alpha T}{T} \right) dT$$

$$\Delta S = m_1\, c_0 \ln\left(T_f / T_1 \right) + m_1\, \alpha \left(T_f - T_1 \right) + m_2\, c_0 \ln\left(T_f / T_2 \right) + m_2\, \alpha \left(T_f - T_2 \right) = 101.67\ J/K$$
